Dec 2

NP- complete problems

Def: X is NP- complete if

(i) $X \in NP$

(ii) $\forall Y \in NP$, $Y \leq_p X$ → Just (ii) $\Rightarrow$ X is NP-hard.

Lemma 1: Let X be an NP- complete problem.

If $X \in P \Rightarrow P = NP$.

Lemma 2: Y is an NP- complete problem. $X \in NP$

If $Y \leq_p X \Rightarrow X$ is also NP- complete.

THM: 3-SAT is NP- complete.

$\Longrightarrow$ COR: IS is NP-complete.

3-SAT $\leq_p$ IS

General strategy to prove X is NP- complete.

Step 1: $X \in NP$ (X = IS)

Step 2: Identify an NP - complete problem Y. (Y = 3-SAT)

Step 3: Prove $Y \leq_p X$. (3-SAT $\leq_p$ IS)

Overview of 3-SAT is NP- complete.

→ Step 1: Define Circuit - SAT

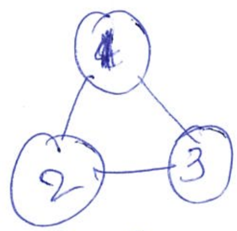
→ Step 2: Prove Circuit - SAT is NP-complete.

Step 3: Circuit SAT $\leq_p$ 3-SAT

k-colorability (k-coloring)

$G = (V, E)$

Def: A k-coloring of G if $c : V \longrightarrow \{1, \dots, k\}$ s.t. $\forall\, (u,w) \in E$, $c(u) \neq c(w)$



← 3-colorable but NOT 2-colorable.

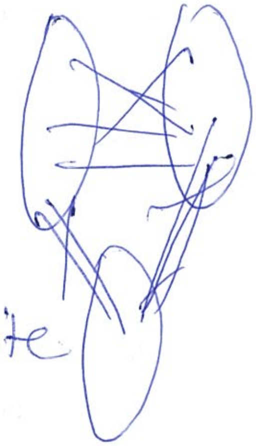
Def: G is k-colorable if $\exists$ a k-coloring for it.

Def: (k-coloring / k-colorability problem) k = 3

i/p: G, k

o/p: T if G is k-colorable

F o/w



Claim1: k-colorability $\in$ NP? 3-partite

Claim2: 2-colorability $\in$ P

THM: 3-SAT $\le_p$ 3-colorability $\le_p$ k-colorability, $k \geq 3$

$\underset{\text{Claim1}}{\Longrightarrow}$ 3-coloring is NP-complete.

HW10 Q3: k-colorability $\le_p$ ~~3~~ SAT.