Gale-Shapley algorithm

n men
n women
2n pref. list

⓪ Initially all men & women are __free__

① In a loop: ← in book: men propose

A __free__ __woman__ proposes to a man

② You have n matched pairs

__Initial state__ : All n men & n women are free

① Let w be a free woman

Q1: Which man m should w propose to?
A1: The man m on top of her pref list.
→ w will propose to m

(Q2) What should m do?

Case 2.1 : m accepts w's proposal
   Issue: m could get a better proposal later on.

Case 2.2 : m rejects w's proposal
   Issue: He might not get a better proposal later on.

→ Case 2.3 : m __conditionally__ accepts
   ⟹ (m, w) are __engaged__

__General state__: All men / women are either __free__ / __engaged__

① All n men & women are engaged
Q2 ⟹ Algo terminates & outputs n engaged pairs as the final output.

ELSE
② ∃ a free woman w?

Q1: Who should w propose to?
A1: propose to best man m she has NOT proposed to yet.
→ w proposes to m ⚥
Q2: What should m do?

Case 2.1: m is free $\Rightarrow$ (m,w) are engaged

Case 2.2: (m,w') are engaged

    Case 2.2.1: $w' > w$ in L_m (no changes)

    Case 2.2.2: $w > w'$ in L_m $\Rightarrow$ (m,w) are engaged
                                    w' free.