# Lecture 19 

CSE 331
Oct 18, 2021

## Please have a face mask on

Masking requirement


LIR requires all students, employees and visitors - regardless of their vaccination status - to wear face coverings while inside campus buildings.

## Project deadlines coming up

| Fri, Oct 29 | Counting inversions $\mathrm{D}^{816} \mathrm{D}^{711} \mathrm{D}^{17} \mathrm{x}^{2}$ | [KT, Sec 5.3] (Project (Problem 1 Coding) in) |
| :---: | :---: | :---: |
| Mon, Nov 1 | Multiplying large integers $\mathrm{D}^{719} \mathrm{D}^{-18} \mathrm{D}^{517} \mathrm{x}^{2}$ | [KT, Sec 5.5] (Project (Problem 1 Reflection) in) |
|  |  | Reading Assignment: Unraveling the mystery behind the identity |
| Wed, Now 3 | Closest Pair of Points $\mathrm{D}^{19} \mathrm{D}^{18} \mathrm{D}^{\text {F17 }} \mathrm{x}^{2}$ | [KT, Sec 5.4] |
| Fri, Nov 5 |  | [ KT , Sec 5.4] (Project (Problem 2 Coding) in) |
| Mon, Nov 8 | Weighted Interval Scheduling $\mathrm{D}^{\mathrm{F} / 9} \mathrm{DF}^{\mathrm{F17}} \mathrm{x}^{2}$ | [KT, Sec 6,1] (Project (Problem 2 Reflection) in) |

## Some other stuff coming up

## What's next?

Now that the miditerms are done, hope Y'ult take some tiece to decomprens Some of you might nave quastions on hew youre doing in the cours, how you did in the mid-term examd ind pertapt tome of you trink yourd like to come ans chat win end.

I jas mated to give Yall sowe hemfer up on tio:

- As a tengemt, note the INW 4 ia ahesby out oaj10
* Or gosi is to be able to finisting grasing footh Thej mid-lems by eally to mid next week.
s Your Thes aho have mid-terms so we approcitin your patience as they prado your mid termel
- Once that is done as with the inWs. ri release the stats as wel as the grading nobric. The urail re-grade polioy will apply fthough keep asap in mind!.
 stind in tre course oumently.
- Il put up a piarsa posi with the dotals once the ferpo. lotior gradso have been assignsd.
- Nole that this will not be the same as the mid-semester grade that I need to sibmit to HuB prainly because the mid terms will not be graded by thas Fridsy. which is when the mid-semestor grades are dul)
* Those who heve a D+ or below in their temporary letter grade. II send emal asking jou to setup a one-on-one mesting fcu10 mins].
- Even if you have a better grade than D. but want to chat about your pertormance, you can albo sign up fut thooe wath Do or befow will get proterence for a alots
- II put up a piacra posi with delals once I finalie the meeting sklas.

```
midtum yraty
```


## Mid-semester grade on HUB

## IGNORE the mid-semester grade on HUB




 youn, which is why I sploaded a onydy mid-semeeter grade for now.

```
nat merm
#n+4
```


# Overheard by a TA (in F19).. 

I can't wait to be done with 331 ......
then I can have a normal life again


Atri Rudra 3:49 PM
Goes for us as well
(3) 4

## Questions?



## Shortest Path problem

Input: Directed graph G=(V,E)
Edge lengths, $\mathrm{I}_{\mathrm{e}}$ for e in E

"start" vertex s in V


Output: Length of shortest paths from $s$ to all nodes in $\vee$

## Towards Dijkstra's algo: part ek

Determine $\mathrm{d}(\mathrm{t})$ one by one

$$
d(s)=0
$$



## Towards Dijkstra's algo: part do

## Determine $\mathrm{d}(\mathrm{t})$ one by one

Let $u$ be a neighbor of $s$ with smallest $\left.\right|_{(s, u)}$

$$
\mathrm{d}(\mathrm{u})=\mathrm{I}_{(\mathrm{s}, \mathrm{u})}
$$



Length of $\sim 0$
Not making any claim on other vertices

## Towards Dijkstra's algo: part teen

## Determine $\mathrm{d}(\mathrm{t})$ one by one

Assume we know $d(v)$ for every $v$ in $R$

Compute an upper bound d'(w) for every w not in R

$$
\begin{aligned}
& d(w) \leq d(u)+l_{(u, w)} \\
& d(w) \leq d(x)+I_{(x, w)} \\
& d(w) \leq d(y)+I_{(y, w)}
\end{aligned}
$$

$$
d^{\prime}(w)=\min _{e=(u, w) \text { in } E, u \text { in } R} d(u)+l_{e}
$$

## Questions/Comments?



## Dijkstra's shortest path algorithm



Input: Directed $G=(\mathrm{V}, \mathrm{E}), \mathrm{I}_{\mathrm{e}} \geq 0$, s in V
$R=\{s\}, d(s)=0$
While there is a $x$ not in $R$ with $(u, x)$ in $E, u$ in $R$
Pick $w$ that minimizes $d^{\prime}(w)$
Add w to R
$d(w)=d^{\prime}(w)$
$d^{\prime}(w)=\min _{e=(u, w) \text { in } E, u \text { in } R} d(u)+l_{e}$
$d(s)=0$
$d(u)=1$
$d(w)=2$
$d(x)=2$
$d(y)=3$
$d(z)=4$


## Questions/Comments?



## Couple of remarks

The Dijkstra's algo does not explicitly compute the shortest paths

Can maintain "shortest path tree" separately

Dijkstra's algorithm does not work with negative weights

Left as an exercise

## Rest of Today's agenda

Prove the correctness of Dijkstra's Algorithm

## Dijkstra's shortest path algorithm

## $P_{u}$ shortest s-u path in "Dijkstra tree"

$$
d^{\prime}(w)=\min _{e=(u, w) \text { in } E, u \text { in } R} d(u)+I_{e}
$$

Input: Directed $G=(\mathrm{V}, \mathrm{E}), \mathrm{I}_{\mathrm{e}} \geq 0$, s in V

```
R={s},d(s)=0
While there is a x not in R with ( }u,x\mathrm{ ) in E, u in R
    Pick w that minimizes d'(w)
    Add w to R
    d(w) = d'(w)
```

Lemma 1: At end of each iteration, if $u$ in $R$, then $P_{u}$ is a shortest s-u path

Lemma 2: If $u$ is connected to $s$, then $u$ in $R$ at the end

## Proof idea of Lemma 1



## Dijkstra's shortest path algorithm

$$
d^{\prime}(w)=\min _{e=(u, w) \text { in } E, u \text { in } R} d(u)+l_{e}
$$

Input: Directed $G=(\mathrm{V}, \mathrm{E}), \mathrm{I}_{\mathrm{e}} \geq 0$, s in V

$\mathrm{O}((\mathrm{m}+\mathrm{n}) \log \mathrm{n})$ time implementation with priority Q

## Reading Assignment

Sec 4.4 of [KT]


