Lecture 19

CSE 331

Oct 18, 2021

Please have a face mask on

Masking requirement



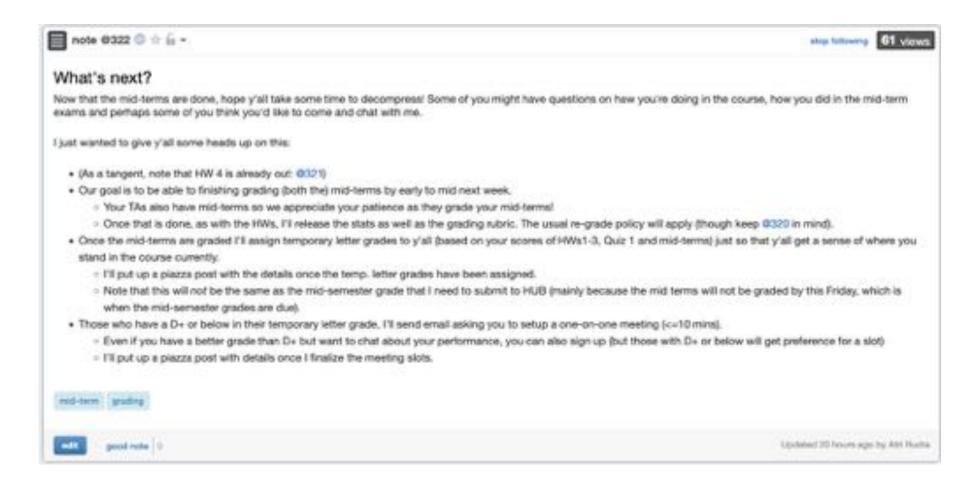
<u>LIR_requires</u> all students, employees and visitors – regardless of their vaccination status – to wear face coverings while inside campus buildings.

https://www.buffalo.edu/coronavirus/health-and-safety/health-safety-guidelines.html

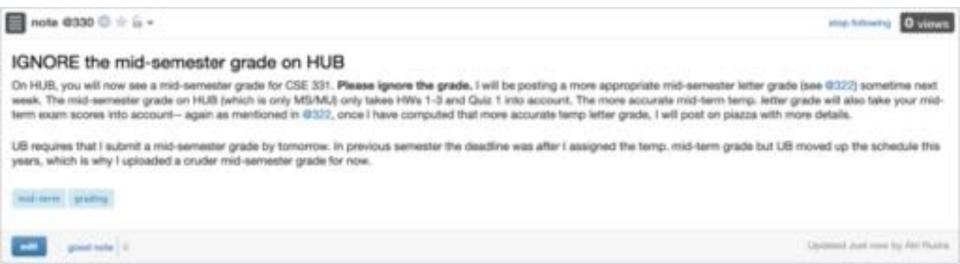
Project deadlines coming up

Fri, Oct 29	Counting Inversions P19 P18 P17 x3	[KT, Sec 5.3] (Project (Problem 1 Coding) in)
Mon, Nov 1	Multiplying large integers □F18 □F17 x²	[KT, Sec 5.5] (Project (Problem 1 Reflection) in) Reading Assignment: Unraveling the mystery behind the identity
Wed, Nov 3	Closest Pair of Points □F19 □F18 □F17 x3	[KT, Sec 5.4]
Fri, Nov 5	Kickass Property Lemma □F19 □F18 □F17 x²	[KT, Sec 5.4] (Project (Problem 2 Coding) in)
Mon, Nov 8	Weighted Interval Scheduling F19 F17 x²	[KT, Sec 6.1] (Project (Problem 2 Reflection) in)

Some other stuff coming up



Mid-semester grade on HUB



Overheard by a TA (in F19)..

I can't wait to be done with 331......

then I can have a normal life again



Questions?

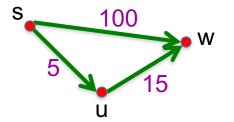


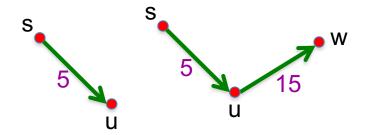
Shortest Path problem

Input: *Directed* graph G=(V,E)

Edge lengths, le for e in E

"start" vertex s in V



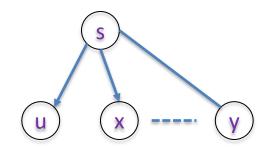


Output: Length of shortest paths from s to all nodes in V

Towards Dijkstra's algo: part ek

Determine d(t) one by one

$$d(s) = 0$$



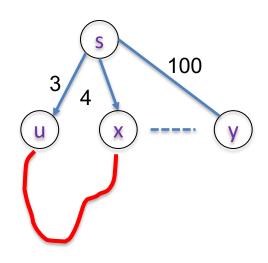
Towards Dijkstra's algo: part do

Determine d(t) one by one

Let u be a neighbor of s with smallest I_(s,u)

$$d(u) = I_{(s,u)}$$

Not making any claim on other vertices



Length of is
$$\ge 0$$

Towards Dijkstra's algo: part teen

Determine d(t) one by one

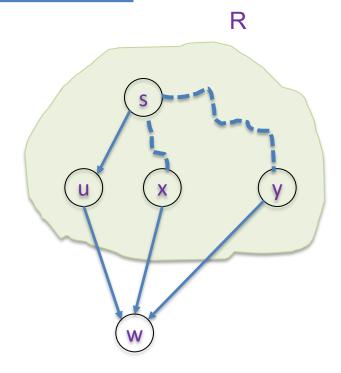
Assume we know d(v) for every v in R

Compute an upper bound d'(w) for every w not in R

$$d(w) \leq d(u) + I_{(u,w)}$$

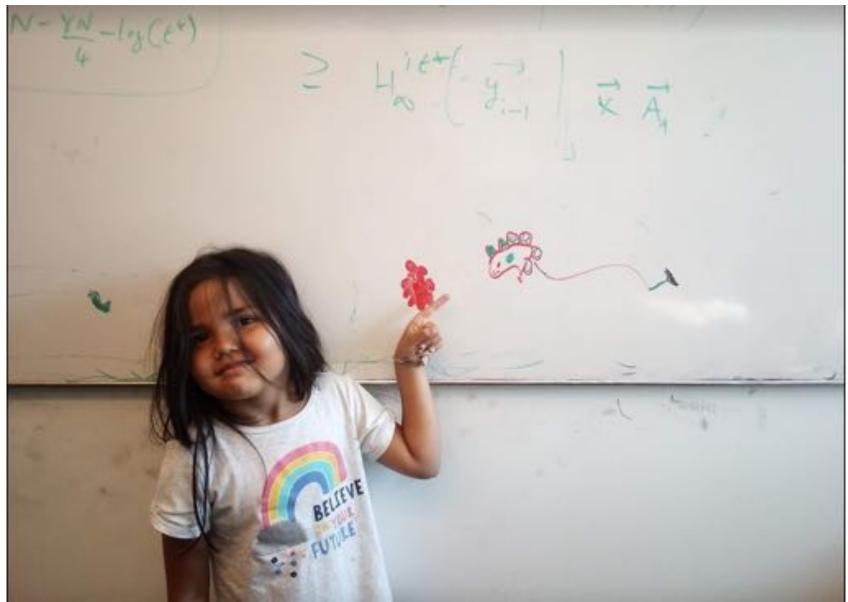
$$d(w) \leq d(x) + I_{(x,w)}$$

$$d(w) \leq d(y) + I_{(y,w)}$$

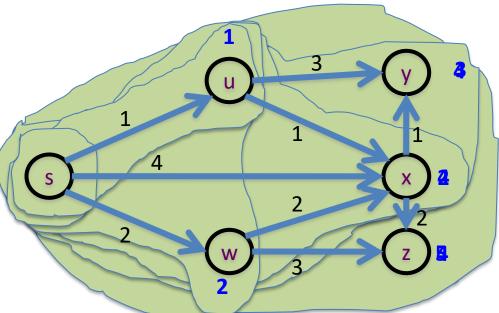


$$d'(w) = \min_{e=(u,w) \text{ in E, u in R}} d(u)+I_e$$

Questions/Comments?



Dijkstra's shortest path algorithm



Input: Directed G=(V,E), $I_e \ge 0$, s in V

$$R = \{s\}, d(s) = 0$$

While there is a x not in R with (u,x) in E, u in R

Pick w that minimizes d'(w)

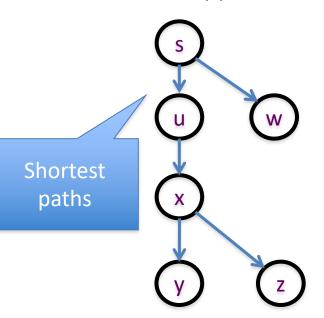
Add w to R d(w) = d'(w)

 $d'(w) = \min_{e=(u,w) \text{ in E, } u \text{ in R}} d(u) + I_e$

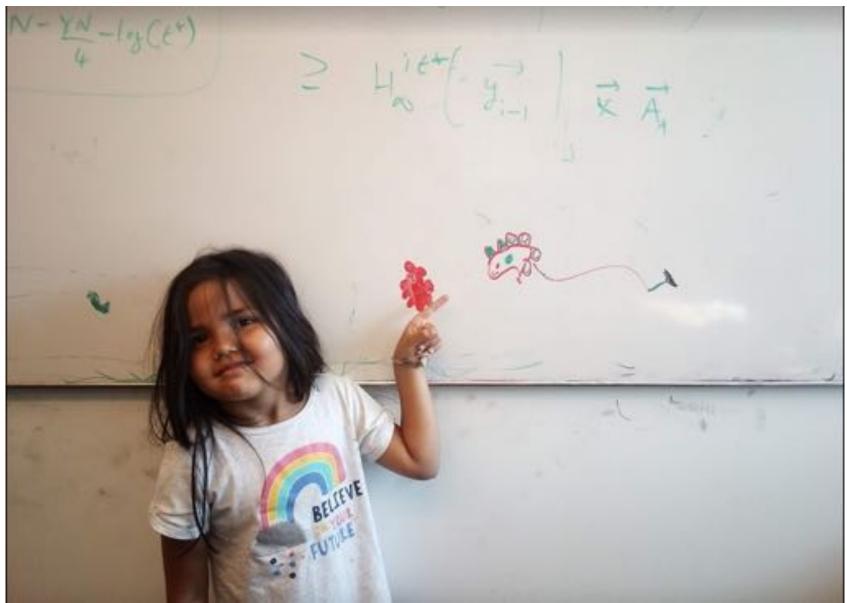
$$d(s) = 0$$
 $d(u) = 1$

$$d(w) = 2$$
 $d(x) = 2$

$$d(y) = 3$$
 $d(z) = 4$



Questions/Comments?



Couple of remarks

The Dijkstra's algo does not explicitly compute the shortest paths

Can maintain "shortest path tree" separately

Dijkstra's algorithm does not work with negative weights

Left as an exercise

Rest of Today's agenda

Prove the correctness of Dijkstra's Algorithm

Dijkstra's shortest path algorithm

P_u shortest s-u path in "Dijkstra tree"

$$d'(w) = \min_{e=(u,w) \text{ in E, u in R}} d(u)+I_e$$

Input: Directed G=(V,E), $I_e \ge 0$, s in V

$$R = \{s\}, d(s) = 0$$

While there is a x not in R with (u,x) in E, u in R

Pick w that minimizes d'(w)

Add w to R

d(w) = d'(w)

Lemma 1: At end of each iteration, if u in R, then P_u is a shortest s-u path

Lemma 2: If u is connected to s, then u in R at the end

Proof idea of Lemma 1



Dijkstra's shortest path algorithm

 $d'(w) = \min_{e=(u,w) \text{ in E, } u \text{ in R}} d(u) + I_e$

Input: Directed G=(V,E), $I_e \ge 0$, s in V

 $R = \{s\}, d(s) = 0$

While there is a x not in R with (u,x) in E, u in R

Pick w that minimizes d'(w)

Add w to R

d(w) = d'(w)

 $\Sigma_{x \in V} O(\ln_x + 1)$ = O(m+n) time

O((m+n)n) time bound is trivial

O((m+n) log n) time implementation with priority Q

At most n iterations

Reading Assignment

Sec 4.4 of [KT]

