# Lecture 24 

CSE 331
Oct 29, 2021

## Please have a face mask on

Masking requirement


LIR requires all students, employees and visitors - regardless of their vaccination status - to wear face coverings while inside campus buildings.

## Coding P1 due TODAY!

| Fri, Oct 29 | Counting inversions $\mathrm{E}^{[16} \mathrm{D}^{614} \mathrm{DF}^{[17} \mathrm{x}^{2}$ | [KT, Sec 5.3] (Project (Problem 1 Coding) in) |
| :---: | :---: | :---: |
| Mon, Nov 1 | Multiplying large integers $\mathbf{D}^{F 19} \mathbf{D}^{F 18} \mathbf{D}^{517} \mathrm{x}^{2}$ | [KT, Sec 5.5] (Project (Problem 1 Reflection) in) Reading Assignment: Unraveling the mystery behind the identity |
| Wed, Now 3 | Closest Pair of Points $\mathrm{D}^{19} \mathrm{Dr}^{18} \mathrm{D}^{\mathrm{F17}} \mathrm{x}^{2}$ | [ KT , Sec 5.4] |
| Fri, Nov 5 | Kickass Property Lemma $\mathbf{D}^{P 19} \mathrm{DV}^{1818} \mathrm{P}^{177} \mathrm{x}^{2}$ | [ KT , Sec 5.4] (Project (Problem 2 Coding) in) |
| Mon, Nov 8 | Weighted Interval Scheduling $\mathrm{D}^{F 17} \mathrm{DP}^{517} \mathrm{x}^{2}$ | [KT, Sec 6,1] (Project (Problem 2 Reflection) in) |

## Group formation instructions

## Autolab group submission for CSE 331 Project

The lowdown on submitting your project (especialy the coding and refection) problerns as a group on Autolab.

Follow instructions


The instruction below are for Coding Problem 1
You will have to repeat the instructions below for EACH ceding AND refiechon protiem on project en Autolab lwth the mpproprane changes to the actuar probieri)
Form your group on Autolab

## Preliminary grading rubric

$\square$ note e401 0 10 $6=$

## Preliminary rubrics for reflections problems up

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Htpul/
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## Preliminary Grading Guidelines

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2. Anseren may not be entirify mievert to the assignmet.

- Levels

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- Exela

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3. They demorutrate that what stabeholders' welie diflers decending on their oen cortast.

## Questions/Comments?



## Rankings



## How close are two rankings?

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## Rest of today's agenda

Formal problem: Counting inversions

Divide and Conquer algorithm

## Problem definition on the board...



## Solve a harder problem

Input: $a_{1}, . ., a_{n}$
Output: LIST of all inversions

$$
\begin{aligned}
& L=\phi \\
& \text { for } i \text { in } 1 \text { to } n-1 \\
& \text { for } j \text { in } i+1 \text { to } n \\
& \text { If } a_{i}>a_{j} \\
& \quad \text { add }(i, j) \text { to } L
\end{aligned}
$$

return L

Optimal for the listing problem

## Example 1: All inversions-- (2i-1,2i)

| 2 | 1 | 3 | 4 | 6 | 5 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Only check (i,i+1) pairs

## Q1: Solve listing problem in $O(n)$ time?

Q2: Recursive divide and conquer algorithm to count the number of inversions?

CountInv (a,n)

$$
\begin{aligned}
& \text { if } n=1 \text { return } 0 \\
& \text { if } n=2 \text { return } a_{1}>a_{2} \\
& a_{L}=a_{1}, . ., a_{[n / 2]} \\
& a_{R}=a_{[n / 2]+1}, . ., a_{n} \\
& \text { return Countlnv }\left(a_{L},[n / 2]\right)+\operatorname{Countlnv}\left(a_{R}, n-[n / 2]\right)
\end{aligned}
$$

## Can be horribly wrong in general

CountInv (a,n)

$$
\begin{aligned}
& \text { if } n=1 \text { return } 0 \\
& \text { if } n=2 \text { return } a_{1}>a_{2} \\
& a_{L}=a_{1}, \ldots, a_{[n / 2]} \\
& a_{R}=a_{[n / 2]+1}, \ldots, a_{n} \\
& \text { return } \operatorname{Countlnv}\left(a_{L},[n / 2]\right)+\operatorname{Countlnv}\left(a_{R}, n-[n / 2]\right)
\end{aligned}
$$

Example where instance has non-zero (can be $\Omega\left(\mathrm{n}^{2}\right)$ ) inversions and algo returns 0 ?


## Bad case: "crossing inversions"

CountInv (a,n)
if $\mathrm{n}=1$ return 0
if $\mathrm{n}=2$ return $\mathrm{a}_{1}>\mathrm{a}_{2}$
$a_{L}=a_{1}, \ldots, a_{[n / 2]}$
$a_{R}=a_{[n / 2]+1}, \ldots, a_{n}$
return $\operatorname{CountInv}\left(\mathrm{a}_{\mathrm{L}},[\mathrm{n} / 2]\right)+\operatorname{Countlnv}\left(\mathrm{a}_{\mathrm{R}}, \mathrm{n}-[\mathrm{n} / 2]\right)$

> Are $a_{L}$ and $a_{R}$ sorted?


## Example 2: Solving the bad case


$a_{L}$ is sorted
First element is $a_{L}$ is larger than first/only element in $a_{R}$
$\mathrm{O}(1)$ algorithm to count number of inversions?
return size of $a_{\llcorner }$

## Example 3: Solving the bad case


$a_{R}$ is sorted
First/only element is $a_{L}$ is smaller than first element in $a_{R}$
$\mathrm{O}(1)$ algorithm to count number of inversions?
return 0

## Solving the bad case

First element of $a_{L}$ is larger than first element of $a_{R}$

## Try to modify the MERGE algorithm



First element of $a_{L}$ is smaller than first element of $a_{R}$


## Divide and Conquer

Divide up the problem into at least two sub-problems

Solve all sub-problems: Mergesort
Recursively solve the sub-problems

Solve stronger sub-problems: Inversions
"Patch up" the solutions to the sub-problems for the final solution

## MergeSortCount algorithm

Input: $a_{1}, a_{2}, \ldots, a_{n}$

Output: Numbers in sorted order+ \#inversion

$$
\begin{aligned}
& T(2)=c \\
& T(n)=2 T(n / 2)+c n \\
& O(n \log n) \text { time }
\end{aligned}
$$

$a_{L}=a_{1}, \ldots, a_{n / 2} \quad a_{R}=a_{n / 2+1}, \ldots, a_{n}$
$\left(c_{L}, a_{L}\right)=$ MergeSortCount $\left(a_{L}, n / 2\right)$
$\left(c_{R}, a_{R}\right)=$ MergeSortCount $\left(a_{R}, n / 2\right)$

Counts \#crossing-inversions+ MERGE
return $\left(c+c_{L}+c_{R}, a\right)$

## MERGE-COUNT $\left(a_{1}, a_{R}\right)$

$$
a_{L}=I_{1}, \ldots, I_{n} \quad a_{R}=r_{1}, \ldots, r_{m}
$$

$$
c=0
$$

$$
i, j=1
$$

while $\mathrm{i} \leq \mathrm{n}$ ' and $\mathrm{j} \leq \mathrm{m}$

$$
\text { if } I_{i} \leq r_{j}
$$

i ++
add $I_{i}$ to output
else

$$
\begin{aligned}
& \text { add } r_{j} \text { to output } \\
& \text { j ++ } \\
& \text { c += n'- } i+1
\end{aligned}
$$



Output any remaining items return c

