

Lecture 27

CSE 331

Nov 5, 2021

Please have a face mask on

























Masking requirement



UR requires all students, employees and visitors – regardless of their vaccination status – to wear face coverings while inside campus buildings.

<https://www.buffalo.edu/coronavirus/health-and-safety/health-safety-guidelines.html>

Coding P2 due TODAY!

Fri, Nov 5	Kickass Property Lemma    x^3	[KT, Sec 5.4] (Project (Problem 2 Coding) in)
Mon, Nov 8	Weighted Interval Scheduling   x^3	[KT, Sec 6.1] (Project (Problem 2 Reflection) in)
Wed, Nov 10	Recursive algorithm for weighted interval scheduling problem   x^2	[KT, Sec 6.1] (HW 6 out)
Fri, Nov 12	Subset sum problem    x^2	[KT, Sec 6.1, 6.2, 6.4]
Mon, Nov 15	Dynamic program for subset sum    x^2	[KT, Sec 6.4]
Wed, Nov 17	Shortest path problem    x^2	[KT, Sec 6.8] (HW 7 out, HW 6 in)
Fri, Nov 19	Bellman-Ford algorithm    x^2	[KT, Sec 6.8]
Mon, Nov 22	The P vs. NP problem  x^2	[KT, Sec 8.1]
Wed, Nov 24	No class	Fall Recess
Fri, Nov 26	No class	Fall Recess
Mon, Nov 29	More on reductions  x^2	[KT, Sec 8.1]
Wed, Dec 1	The SAT problem  x^2	[KT, Sec 8.2] (HW 8 out, HW 7 in)
Fri, Dec 3	NP-Completeness  x^2	[KT, Sec. 8.3, 8.4] (Project (Problem 3 Coding) in)
Mon, Dec 6	k-coloring problem  x^2	[KT, Sec 8.7] (Quiz 2) (Project (Problem 3 Reflection) in)

Group formation instructions

Autolab group submission for CSE 331 Project

The lowdown on submitting your [project](#) (especially the [coding](#) and [reflection](#)) problems as a group on Autolab.

Follow instructions **EXACTLY** as they are stated

The instructions below are for Coding Problem 1

You will have to repeat the instructions below for EACH coding AND reflection problem on project on Autolab (with the appropriate changes to the actual problem).

Form your group on Autolab

Groups on Autolab will NOT be automatically created

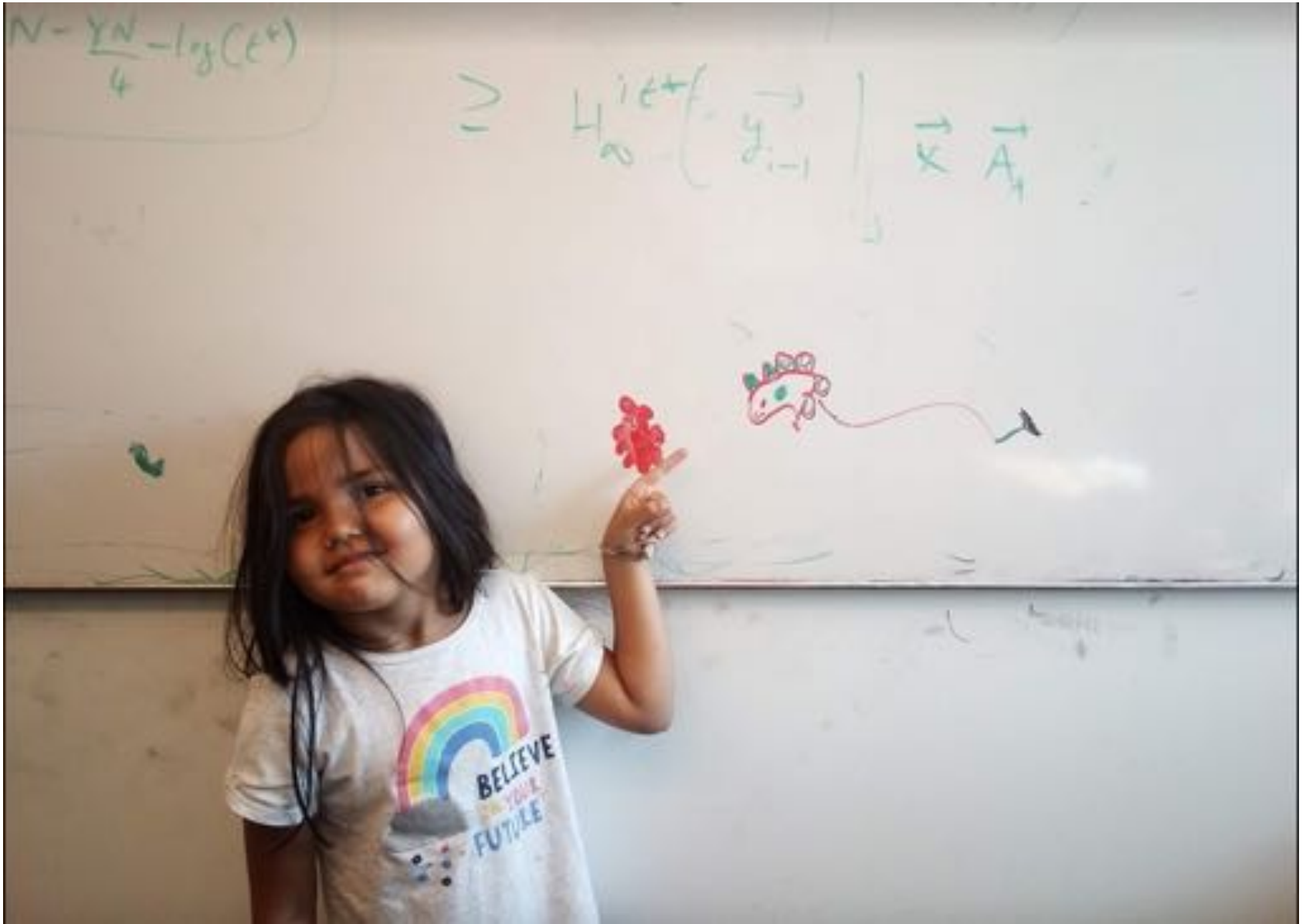
You will have to form a group on Autolab by yourself (as a group). Read on for instructions on how to go about this.

[Click to add notes](#)

Have fun @ UB Hacking!



Questions/Comments?

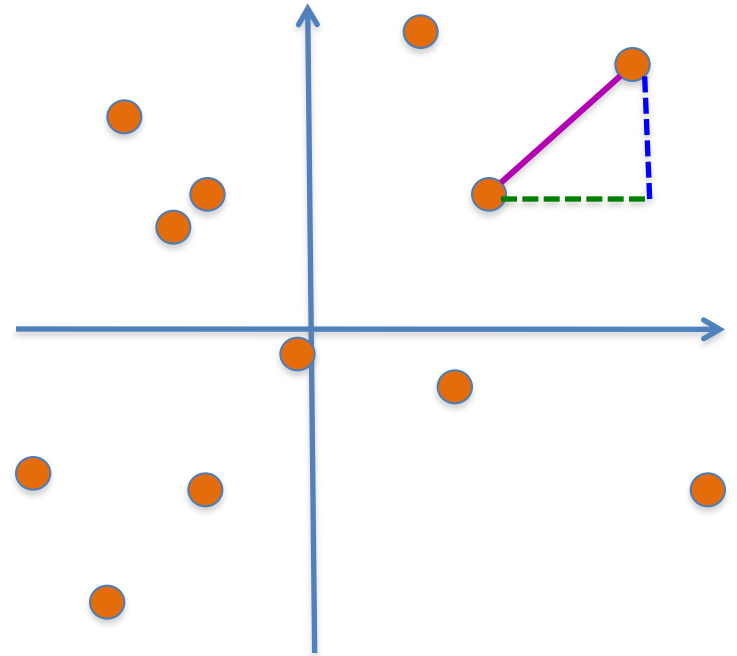


Closest pairs of points

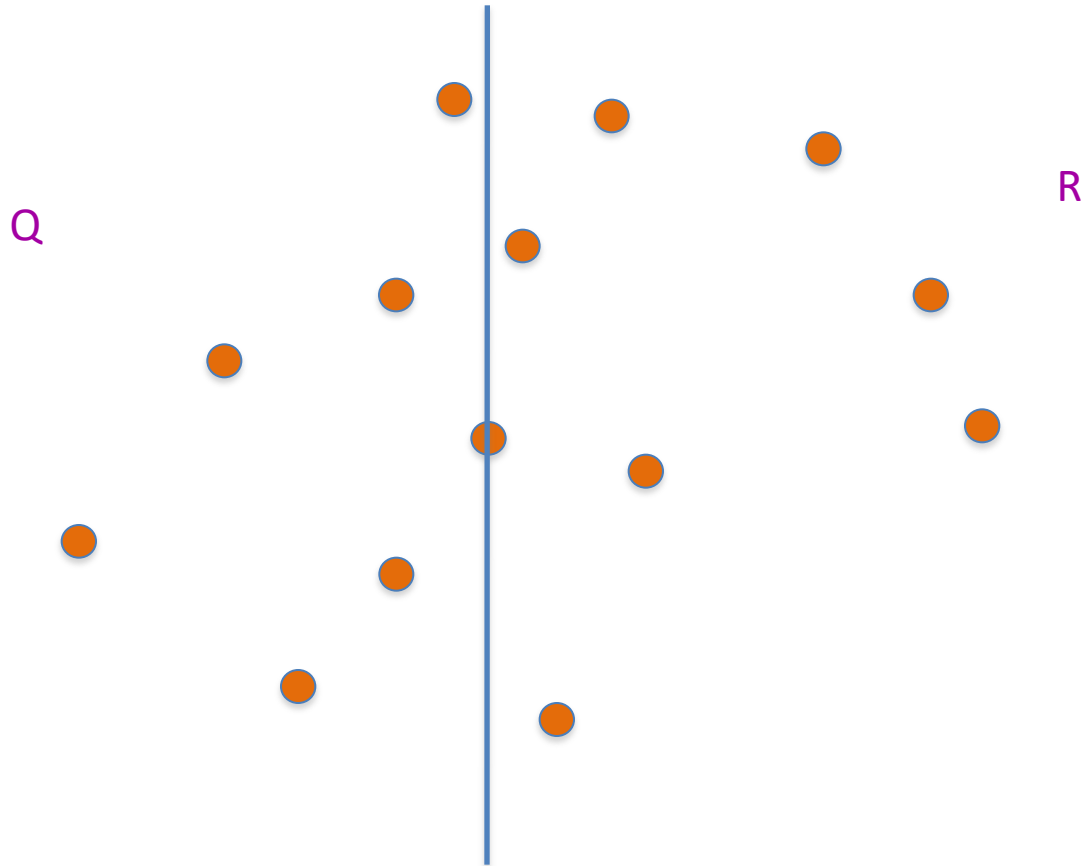
Input: n 2-D points $P = \{p_1, \dots, p_n\}$; $p_i = (x_i, y_i)$

$$d(p_i, p_j) = ((x_i - x_j)^2 + (y_i - y_j)^2)^{1/2}$$

Output: Points p and q that are closest

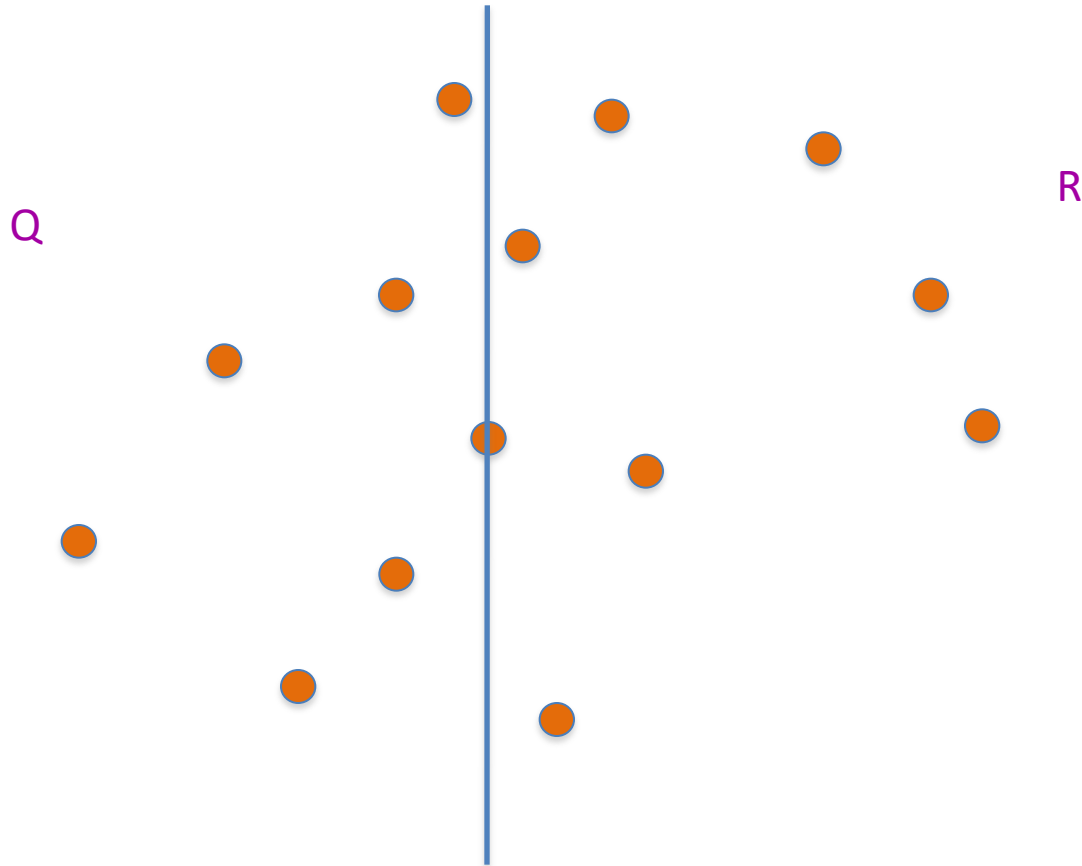


Dividing up P



First $n/2$ points according to the x -coord

Recursively find closest pairs



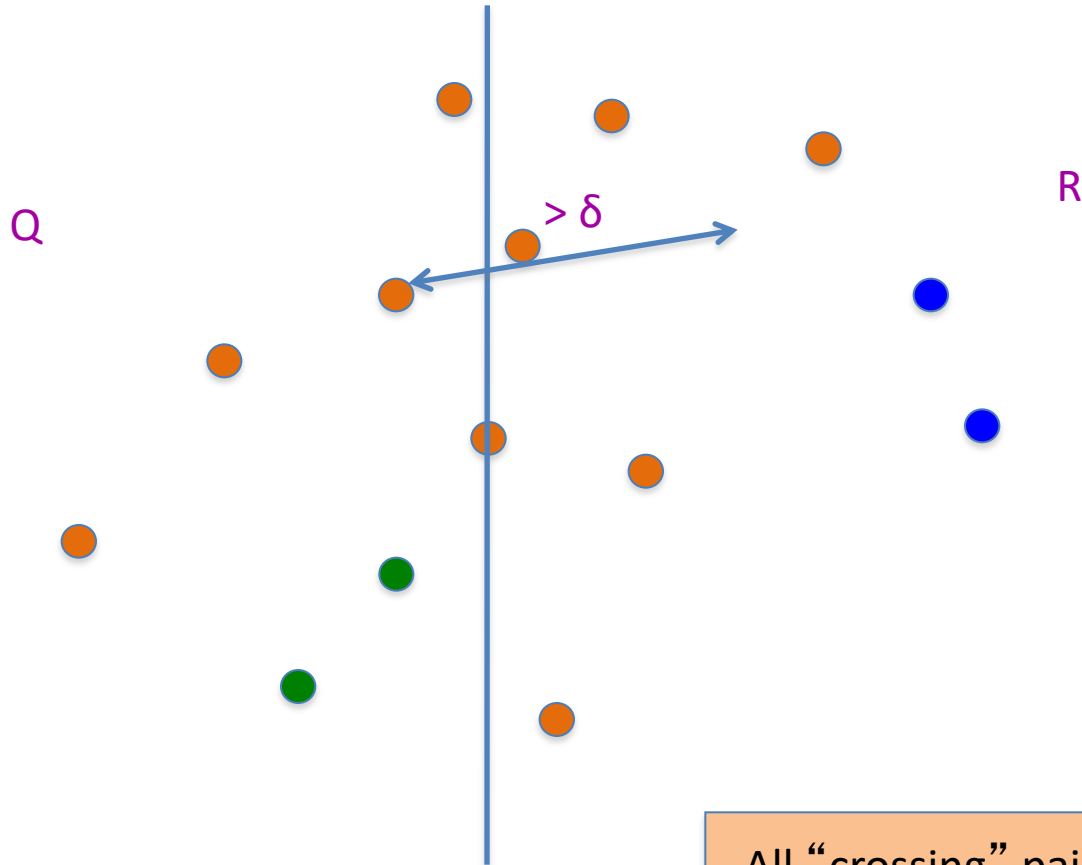
$$\delta = \min(\text{blue}, \text{green})$$

An aside: maintain sorted lists

P_x and P_y are P sorted by x -coord and y -coord

Q_x, Q_y, R_x, R_y can be computed from P_x and P_y in $O(n)$ time

An easy case

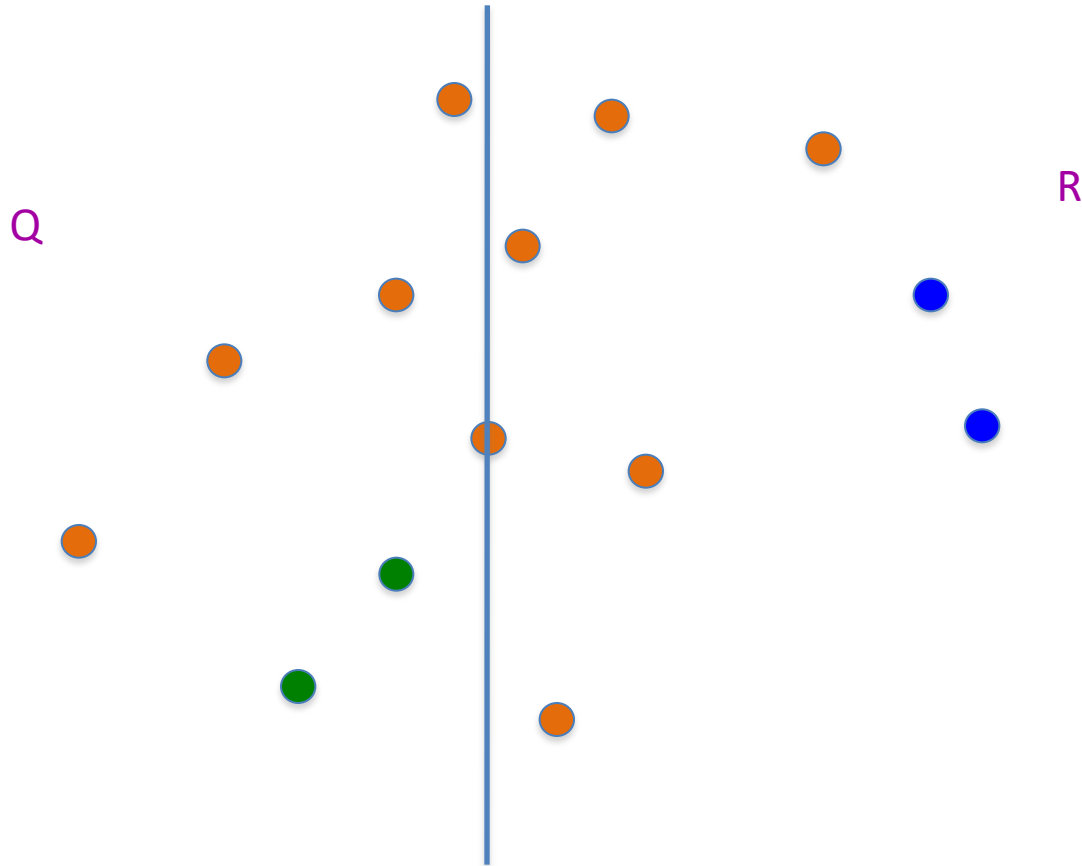


All “crossing” pairs have distance $> \delta$

$\delta = \min(\text{blue, green})$



Life is not so easy though



$$\delta = \min(\text{blue}, \text{green})$$

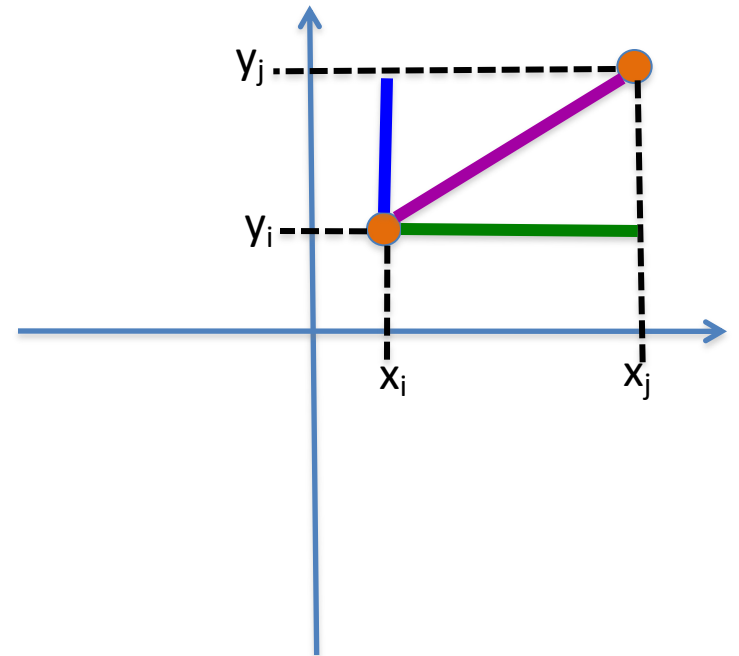
Questions/Comments?



Euclid to the rescue (?)

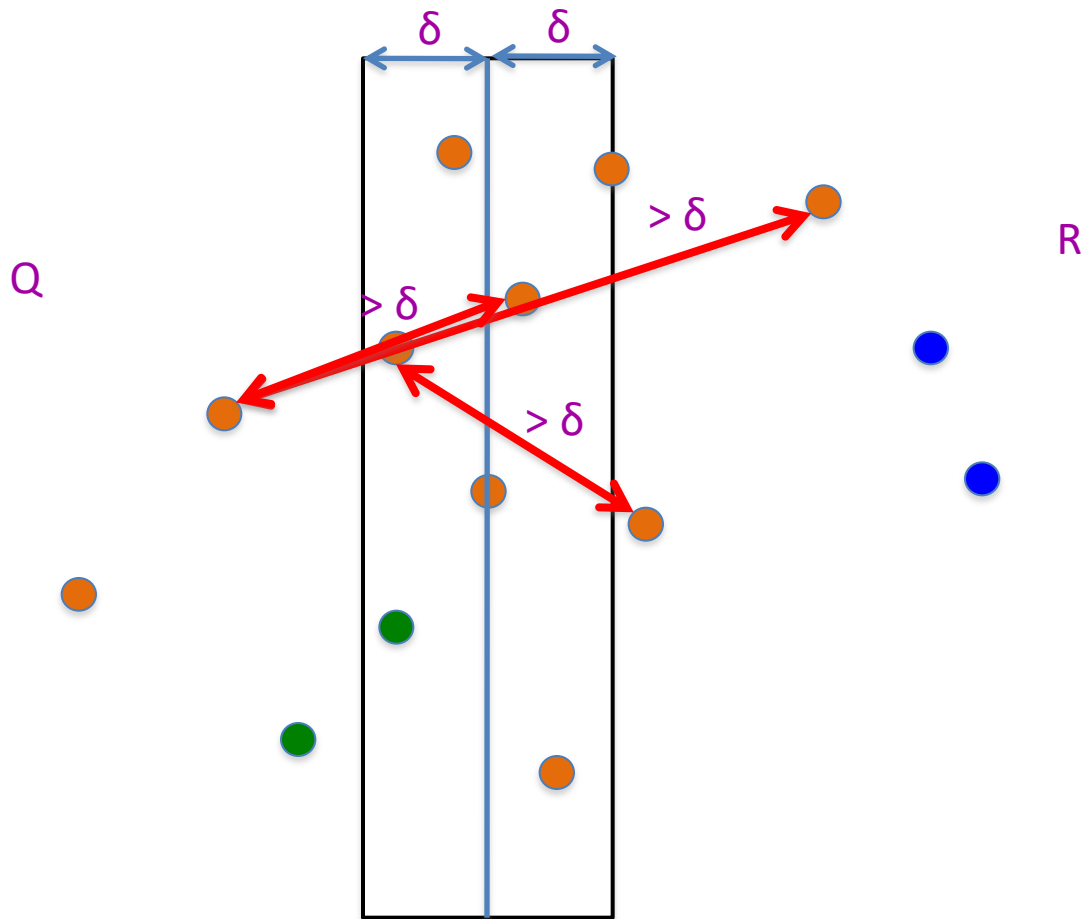


$$d(p_i, p_j) = ((x_i - x_j)^2 + (y_i - y_j)^2)^{1/2}$$



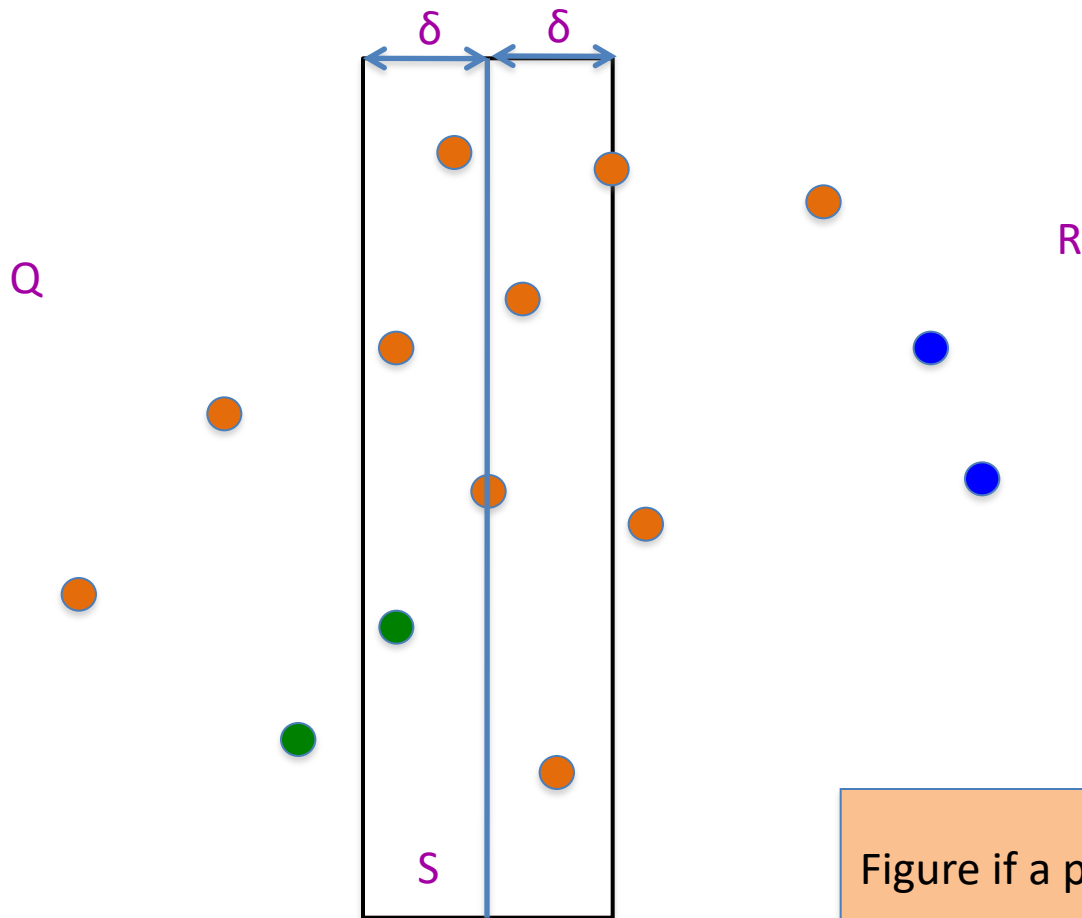
The **distance** is larger than the **x** or **y**-coord difference

Life is not so easy though



$$\delta = \min(\text{blue}, \text{green})$$

All we have to do now



$$\delta = \min(\text{blue}, \text{green})$$

Figure if a pair in S has distance $< \delta$

The algorithm so far...

Input: n 2-D points $P = \{p_1, \dots, p_n\}$; $p_i = (x_i, y_i)$

$O(n \log n) + T(n)$

Sort P to get P_x and P_y

Closest-Pair (P_x, P_y)

$O(n \log n)$

$T(< 4) = c$

If $n < 4$ then find closest point by brute-force

Q is first half of P_x and R is the rest

$O(n)$

$T(n) = 2T(n/2) + cn$

Compute Q_x, Q_y, R_x and R_y

$O(n)$

$(q_0, q_1) = \text{Closest-Pair}(Q_x, Q_y)$

$(r_0, r_1) = \text{Closest-Pair}(R_x, R_y)$

$\delta = \min(d(q_0, q_1), d(r_0, r_1))$

$O(n)$

$S = \text{points } (x, y) \text{ in } P \text{ s.t. } |x - x^*| < \delta$

$O(n)$

return **Closest-in-box** ($S, (q_0, q_1), (r_0, r_1)$)

Assume can be done in $O(n)$

$O(n \log n)$ overall

Rest of today's agenda

Implement Closest-in-box in $O(n)$ time