#### Lecture 5

CSE 331 Sep 10, 2019

#### Please have a face mask on

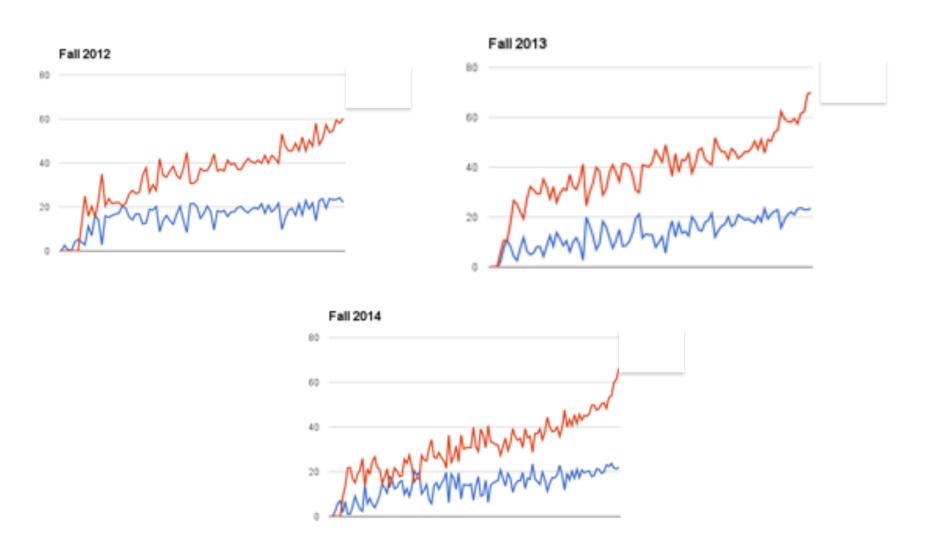
#### Masking requirement



<u>LIR\_requires</u> all students, employees and visitors – regardless of their vaccination status – to wear face coverings while inside campus buildings.

https://www.buffalo.edu/coronavirus/health-and-safety/health-safety-guidelines.html

## Can you guess the correlation?



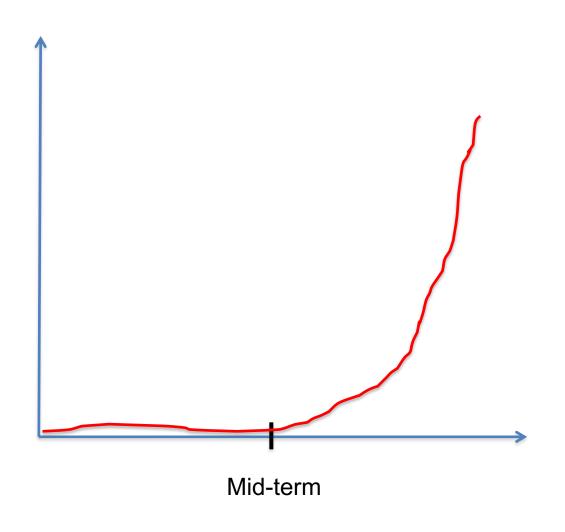
#### Another comment

#### Discomfort with proofs

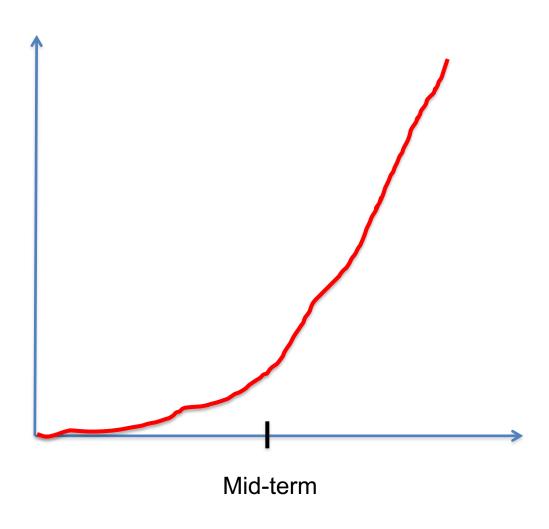
I will not cover proof basics in class anymore

Please read support pages and some utilize (next few) Office hours!

# Lecture pace (till Fall 18)

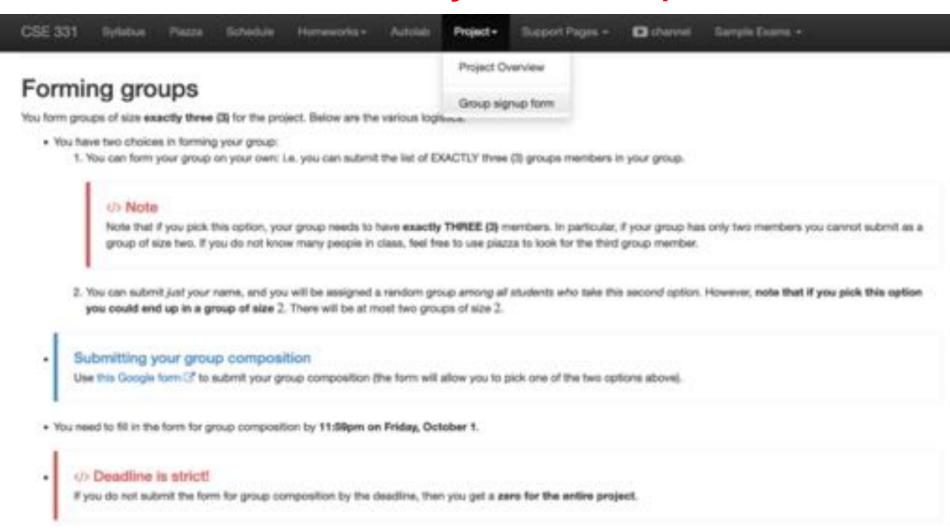


## Lecture pace

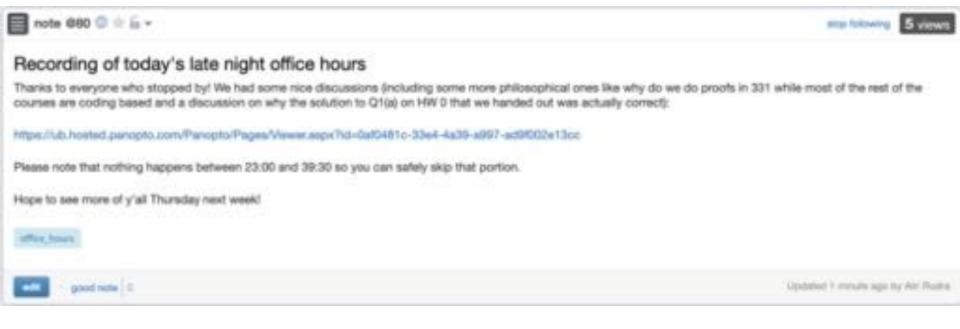


#### Register your project groups

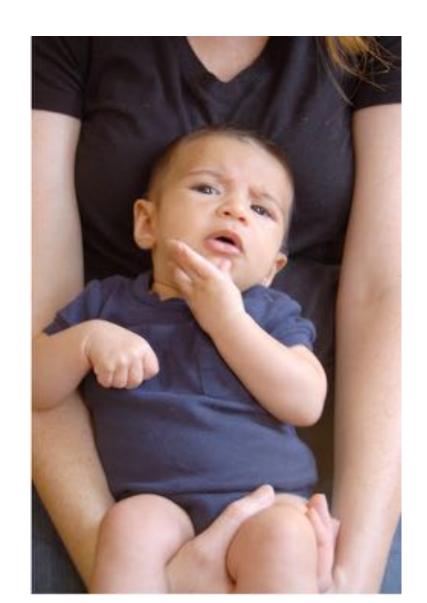
Deadline: Friday, Oct 1, 11:59pm



#### Do come to late night OH!



# Questions/Comments?



## (Perfect) Matching

A matching  $S \subseteq M \times W$  such that following conditions hold:

S is a **set** of pairs (m,w) where m in M and w in W

#### exactly

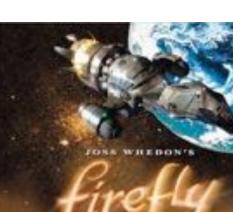
- (1) For every woman w in W, exist at most one m such that (m,w) in S exactly
- (2) For every man m in M, exist at most one w such that (m,w) in S

Perfect matching

## On matchings

Mal







Inara

Wash

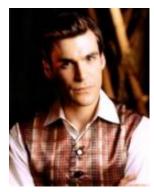






Zoe

Simon





Kaylee

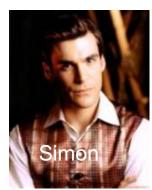
# A valid matching













## Not a matching













## Perfect Matching













#### **Preferences**























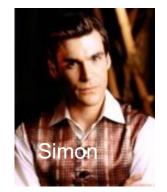


















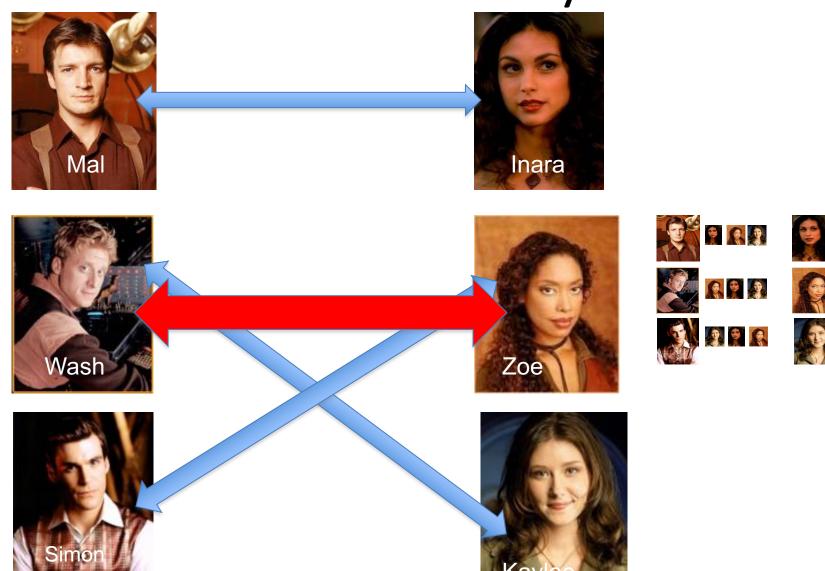








## Instability



## A stable marriage

Even though BBT and JA are not very happy







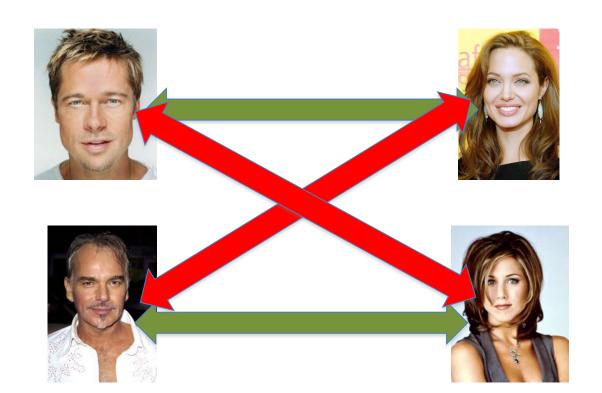




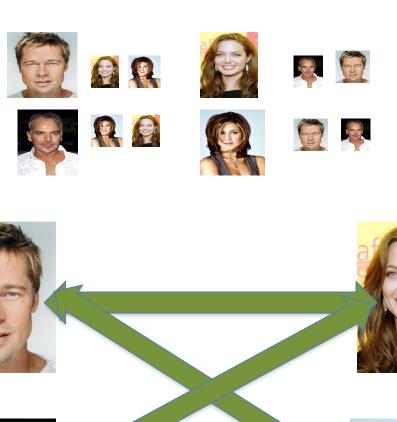


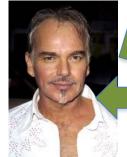






## Two stable marriages







#### Stable Marriage problem

Set of men M and women W

Preferences (ranking of potential spouses)

Matching (no polyandry/gamy in M X W)

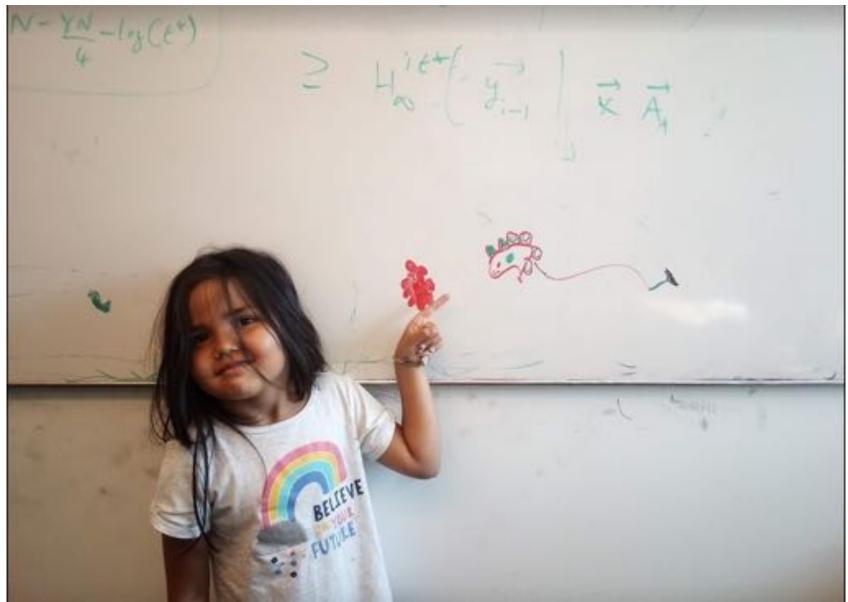
Perfect Matching (everyone gets married)

**Instablity** 

**Input:** M and W with preferences **Output:** Stable Matching

Stable matching = perfect matching+ no instablity

## Questions/Comments?



#### **Two Questions**

Does a stable marriage always exist?

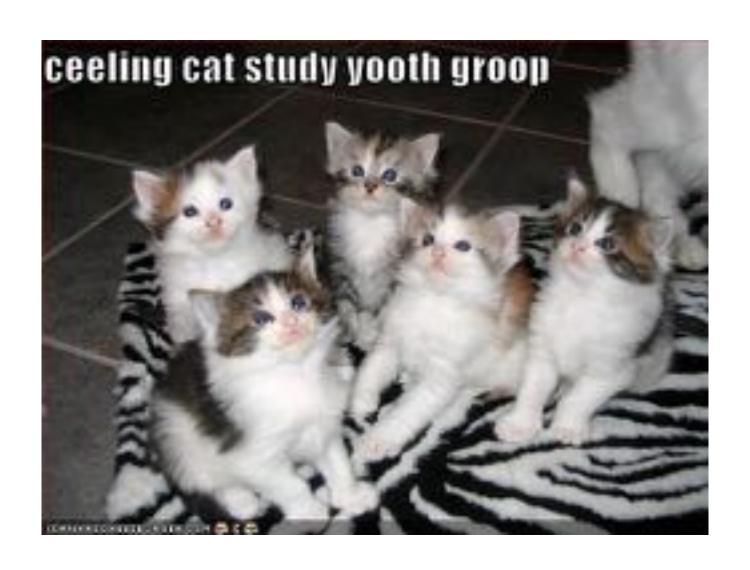
If one exists, how quickly can we compute one?

## Today's lecture

Naïve algorithm

Gale-Shapley algorithm for Stable Marriage problem

## Discuss: Naïve algorithm!



## The naïve algorithm

Incremental algorithm to produce all n! prefect matchings?

Go through all possible perfect matchings S

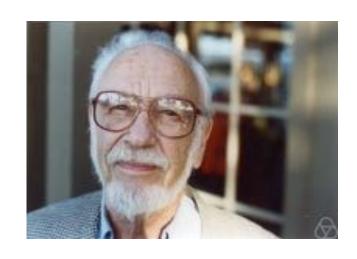
If S is a stable matching

then Stop



Else move to the next perfect matching

## Gale-Shapley Algorithm



David Gale



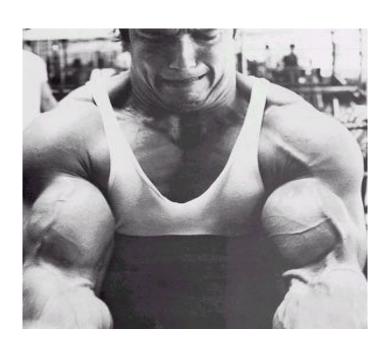
**Lloyd Shapley** 

O(n<sup>2</sup>) algorithm

## Moral of the story...







# Questions/Comments?



## Rest of today's agenda

Gale Shapley (GS) algorithm

Run of GS algorithm on an instance