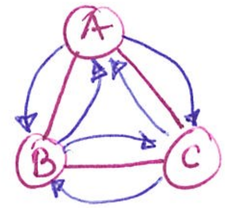
Sep 22

Graphs

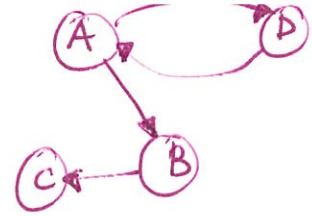
$G = (V, E)$ $E \subseteq V \times V$

set of vertices → V; set of edges → E



$V = \{A, B, C\}$

$E = \{(A,B), (B,C), (C,A), (B,A), (C,B), (A,C)\}$



$V = \{A, B, C, D\}$

$E = \{(A,B), (B,C), (A,D), (D,A)\}$

$n = 4$

$m = 4$

Defaults: $|V| = n$; $|E| = m$

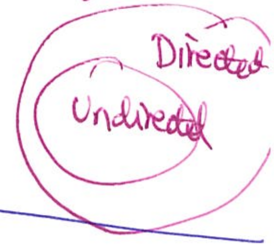
Def: G is undirected $\Longleftrightarrow$ $\forall\, u \neq w \in V$, $(u,w) \in E \Longleftrightarrow (w,u) \in E$

Q! (•) Airline map (undirected) (•) Wikipedia articles (directed)

Default: G is undirected.

Claim: Every undirected graph is also a directed graph.

Pf idea: replace every



Paths

D, C, B, A ✓ 3

A, B, C, D ✓ 3

A, B, C, B ✓ 3

A, C, D ✗ (directed)

D, C, B, A ✗

A, B, C, D ✓

A, B, C, B ✗

A, C, D ✗

Def: A path in $G = (V, E)$ is a sequence of vertices $u_1, \ldots, u_k$ { u_1 — u_k path } s.t. $\forall\, i \in [k-1] = \{1, \ldots, k-1\}$

$(u_i, u_{i+1}) \in E$

Notes: (i) u_i need not be distinct (ii) holds for directed G

Def: A simple path is a path w/ no repeated vertices

Ex: Any simple path has <u>length</u> $\leq n-1$.

Def: The length of a path is the number of <u>edges</u> in it

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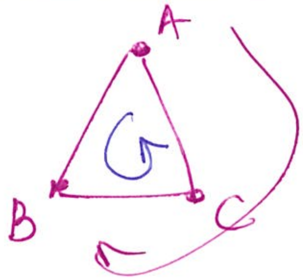
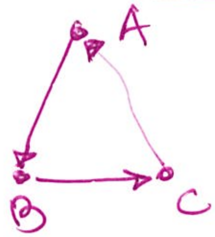
Cycles:



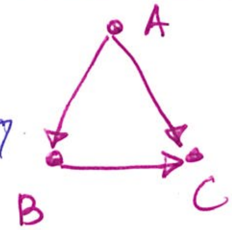
A, C, B, A ✓

A, B, C, A



A, C, B, A ✗

A, B, C, A ✓



A, C, B, A ✗

A, B, C, A ✗

Directed Acyclic graph (DAG)

Def: A cycle is a sequence

$u_1, u_2, \dots, u_k = u_1$ $\quad u_1, \dots, u_{k-1}$ are distinct

$\forall$ ~~i∈~~ ~~x∈~~ $i \in [k-1]$, $(u_i, u_{i+1}) \in E$