Sep 27

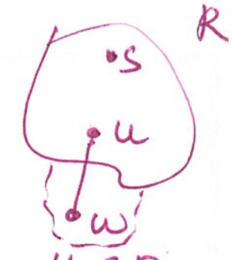
EXPLORE (s)

0. $R \leftarrow \{s\}$

1. While $\exists\ (u,w) \in E$ s.t. (such that) $w \notin R$, $u \in R$

   Add w to R

2. Output $R^* \leftarrow R$



THEOREM: For all G, start vertices s, $R^* = CC(s)$

$\Rightarrow$ COROLLARY: BFS is correct!

General idea: to show $A = B \Leftrightarrow A \subseteq B$ AND $B \subseteq A$

Lemma 1: $R^* \subseteq CC(s)$ ← everything that is output by Explore is correct (in CC(s))

Lemma 2: $CC(s) \subseteq R^*$ ← everything that should be output (ie in $CC(s)$) is output by Explore.

Lemma 1+2 $\Rightarrow$ THM ■

Pf. of Lem 1: Ex. (by induction)

Pf (idea) of Lemma 2: Pf. by contradiction

Note: Since R^* is output by Explore $\Rightarrow$ Explore has terminated — (#)

Assume $CC(s) \not\subseteq R^*$

$\Rightarrow \exists\ w \in CC(s)$ BUT $w \notin R^*$

$\Leftrightarrow \exists$ s–w path P in G but $w \notin R^*$

Since P starts inside of R^* (as $s \in R^*$)
but ends up outside of R^* (as $w \notin R^*$)

$\Rightarrow P$ has to "cross" the boundary of R^* at some point

$\Rightarrow \exists\ (x,y) \in E$ s.t. $x \in R^*$, $y \notin R^*$

$\Rightarrow y$ should have been added to R by Explore

$\Rightarrow$ Explore should not have terminated $\Rightarrow$ contradicts (#) ■