Oct 15

Last Friday: Special case $l_e = 1$ $\forall e \in E$ ($= L$ for the same L)

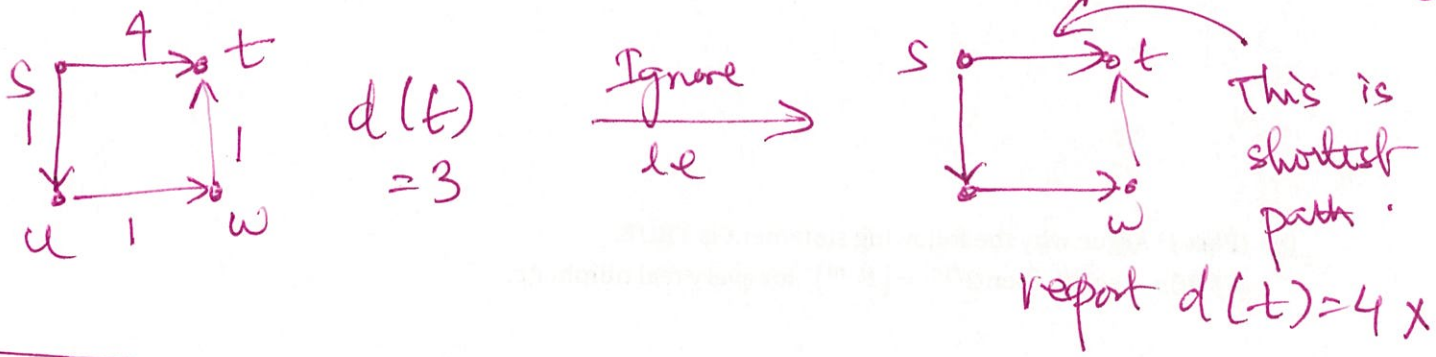
→ run HW3 Q3 algo.

General case: $l_e \geq 0$ $\forall e \in E$ (but l_e can be different)

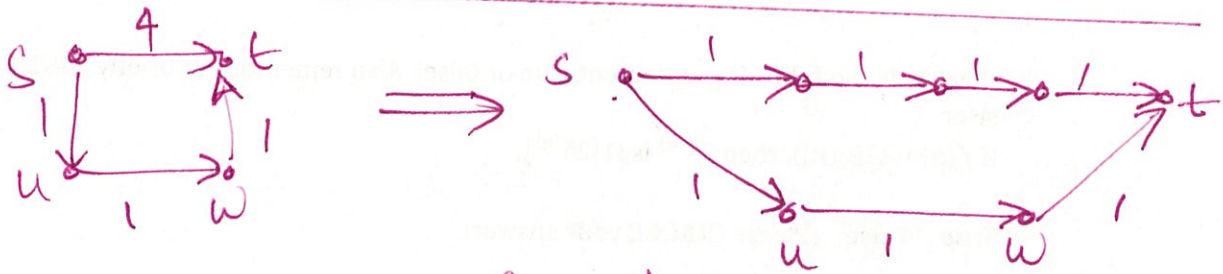
↑ integers

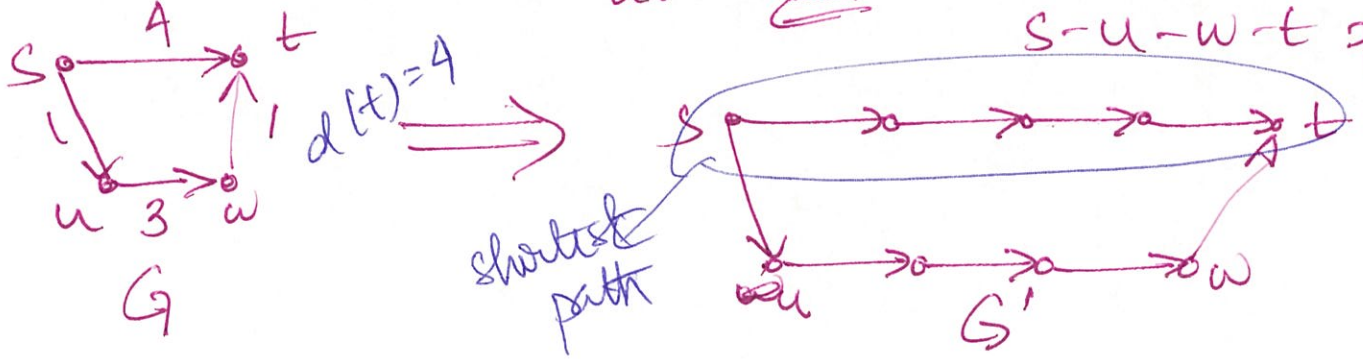
Algo idea: Reduce the general case to $l'_e = 1$ $\forall e \in E$

✗ Idea 1: Ignore all l_e values i.e. assume $l_e = 1$ $\forall e \in E$



Idea 2:





$\Leftarrow$ s.t. HW 3 Q3 algo pick

Algo idea: Replace an edge e with a path of length l_e. ← (new paths do not share any nodes/edges)

Claim: A shortest s-t path in G ⟺ equiv shortest s-t path in G'. ← Ex.

⇒ Run HW3 Q3 on G' ← runs in $O(|V'|+|E'|)$

Correctness: Claim + correctness of HW3 Q3 algo.

Runtime analysis $l_{max} = \max_{e \in E} l_e$ $G' = (V', E')$

$n' = |V'|$, $m' = |E'|$

Runtime: $O(m'+n')$

$m' \leq l_{max}\, m$

$n' \leq l_{max}\, n$

$= O(l_{max}(m+n))$

If $l_{max} = O(1)$ then fine

But $l_{max} = n^{100}$ → runtime $O(n^{100}(n+m))$

Recap/Aside: RAM model → unit of space is a register

If you have n input items → each register $O(\log n)$ bits

→ for adj list repr: $O(m+n)$ registers

→ $O(n^{100}(n+m))$ runtime in RAM model.

Q: How many registers do you need to represent $l_e \leq n^{100}$

$O(\log n)$ bits $= O(1)$ registers

$\approx 100 \log n$ ≈ 100 registers

IDEALLY: have $O(m+n)$ runtime for the case $l_{max} \leq n^{O(1)}$