Oct 18

Lemma 1: At the end of each iteration, if $u \in R$, the path P_u is a shortest s-u path.

(s-u path in Dijkstra tree)

Pf (idea): By induction on $|R|$

Base case: $|R| = 1 \Rightarrow R = \{s\}$, $d(s) = 0$ ✓

I.H.: Assume lemma is true for $|R| = k$ ($k \geq 1$)

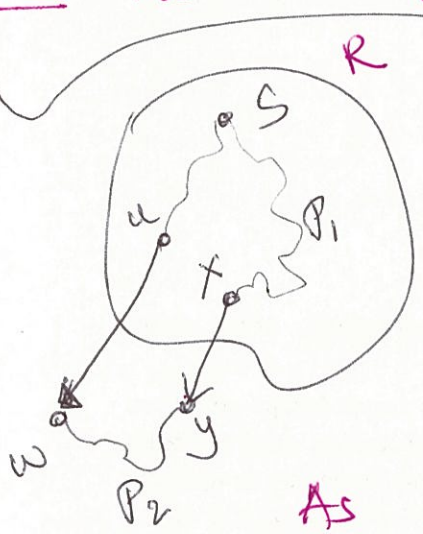
I.S.: Assume $|R| = k+1$

Let w be the $(k+1)^{th}$ vertex added to R

assume w was discovered from u

$\{ d(w) = d(u) + \ell(u,w) \}$ our construction of Dijkstra tree

Goal: P_w is a shortest s-w path. $P_w = P_u, w$



Pf (idea) By contradiction.

Assume $\exists$ an s-w path $P'_w \neq P_w$

but $\ell(P'_w) < \ell(P_w)$ ——— (*)

As $s \in R$ but $w \notin R \Rightarrow P'_w$ "crosses" R at some edge (x, y)

$P'_w = P_1, x, y, P_2$

$d'(w) = \min_{\substack{(u,w) \in E \\ u \in R}} \{ d(u) + \ell_{(u,w)} \}$

$\ell(P_2) \geq 0$

$\ell(P'_w) = \ell(P_1) + \ell_{(x,y)} + \ell(P_2)$ def. of $d(y)$

$\geq d(x) + \ell_{(x,y)} + \ell(P_2)$ ($d(x)$ is distance from s to x)

$\geq d'(y)$

$\geq d'(y) + \ell(P_2) \geq d'(y)$

$\geq d'(w)$ (Dijkstra chose w over y)

$= \ell(w)$ (Algo def)

$= \ell(P_w)$

$\Rightarrow \ell(P'_w) \geq \ell(P_w) \Rightarrow$ contradicts (*) ■