Nov 5

KICKASS PROPERTY LEMMA

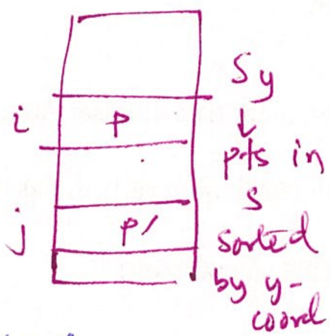
For every $p \neq p' \in S$

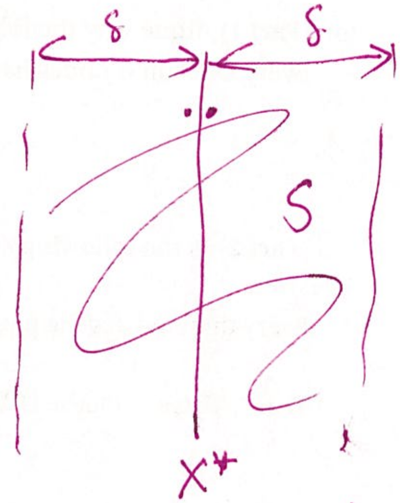
s.t $d(p, p') < \delta$

if $S_y[i] = p$

$S_y[j] = p'$

then $|i-j| \leq 15 (!)$





Note: "15" can be made to be "9" (even as small as 7).

$S = \{(x,y) \in P \mid |x - x^*| < \delta\}$

$x^* - \delta < x < x^* + \delta$

Q: O(n) algo for Closest-in-box?

$|S| = n'$

for $i = 1 \ldots n'-1$ ← ≤ n times — O(n)

check — O(1)

$(S_y[i], S_y[i+1]), (S_y[i], S_y[i+2])$

$\ldots, (S_y[i], S_y[i+15])$ → $\min(n', i+15)$

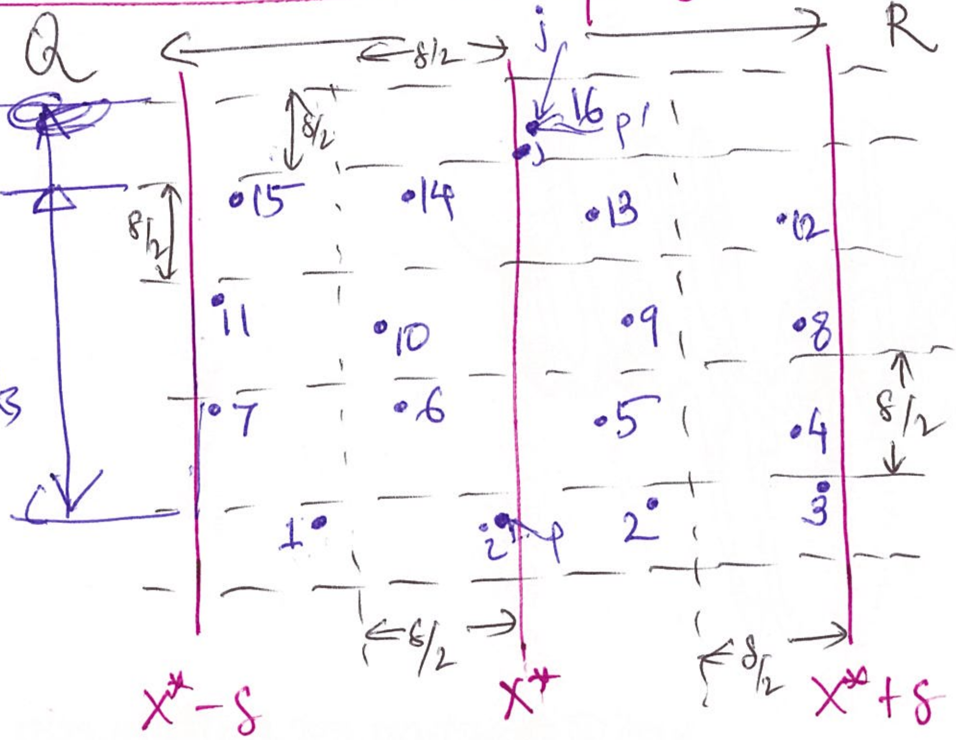
Let (p_i, p_i') be closest pair of pts

Let (p, p') be closest pair of points among $(p_1, p_1'), \ldots, (p_{n'-1}, p'_{n'-1})$ — O(n)

If $d(p, p') < \delta$
    return (p, p')
else
    return NULL — O(1)

overall: O(n) time

Pf (idea) of Kickass Property Lemma



$d(p,p') \geq 3\delta/2 > \delta$

$\Rightarrow$ contradicts (*) ▣

(*) Assume $d(p,p') < \delta$

$S_y[i] = p$

$S_y[j] = p'$

wlog $j \geq i+16$

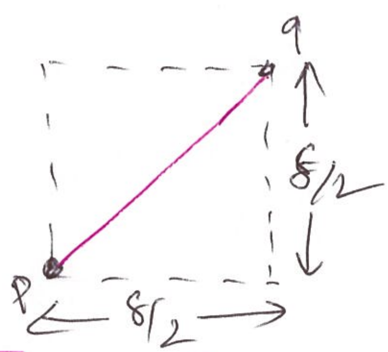
$i, i+1, i+2, \ldots i+15, i+16$ → j

Claim: Every $\frac{\delta}{2} \times \frac{\delta}{2}$ square has <u>atmost one</u> point p from S.

<u>Pf (idea)</u>: By contradiction.

Assume $\exists$ points p & q inside one $\frac{\delta}{2} \times \frac{\delta}{2}$ square

<u>Ex!</u> p & q are farthest apart if on diagonal of square



$$d(p,q) = \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2}$$

$$= \sqrt{\frac{\delta^2}{4} + \frac{\delta^2}{4}} = \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}} < \delta.$$

$\Rightarrow$ contradicts def of δ (= $\min(d(q_0,q_1), d(r_0,r_1))$)

(as every $\frac{\delta}{2} \times \frac{\delta}{2}$ square is in Q or R completely) ▣