Sep 20

Implementing GS

Initialization ← $\leq T_0$

while (..) ← # iteration $= T_1 \leq n^2$

Body ← $\leq T_2$

Output S ← $\leq T_3$

Overall runtime: $\leq T_0 + T_1 \cdot T_2 + T_3$

$\leq T_0 + n^2 \cdot T_2 + T_3$

$\leq O(n^2) + n^2 \cdot O(1) + O(n^2)$

$\leq O(n^2) + O(n^2) + O(n^2) \leq O(n^2)$.

If we can argue
(i) $T_0, T_3 \leq O(n^2)$
(ii) $T_2 \leq O(1)$

Notation change: $M = [n] \stackrel{\text{def}}{=} \{1, .., n\}$

$W = [n]$

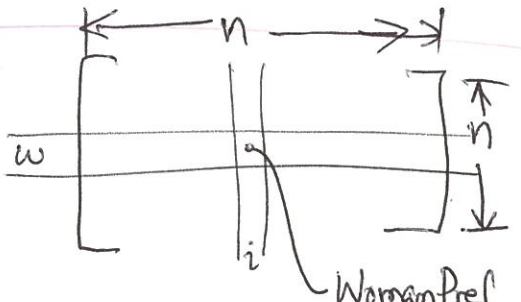
$\{w_1, .., w_n\} \mapsto \{1, .., n\}$

$\{m_1, .., m_n\} \mapsto \{1, .., n\}$

⇒ Array indices start at 1

Q0) How is the input represented?

2D-arrays : WomanPref, ManPref



WomanPref[w][i] = ID of the i^{th} most prefered man in L_w

ManPref[m][i] = ID of woman in L_m

Initialization: ~~n/a~~ —

Query: Read a specific location : $O(1)$

Update: ~~n/a~~ —

Q1) How to find a free woman w?

A1) Maintain a linked list free (of free women)

Init: Add all n women to free : O(n)

Query: Pick # (say) the 1st woman w in free (+ delete w from free) ← O(1)

Update: (w proposed to m)

Case 1: m was free → (do nothing)

Case 2.1 (m, w') remains engaged → Add w to free

Case 2.2 (m, w) get engaged → Add w' to free

} O(1)

Q2) How do we figure out w's best unproposed man m?

A2) Maintain array next of length n

next[w] = rank of the man that she should propose to.

Init: Next[w] = 1 ∀w : O(n)

Query: Who should w propose to? O(1) → WomanPref[w][next[w]]

Update: Next[w]++ } O(1) = Next[w] = Next[w]+1

Q3) ~~How do we figure out w's best unproposed man~~?

How do we figure who m is engaged to?

A3) Array current of length n

$$Current[m] = \begin{cases} -1 & \text{if } m \text{ is free} \\ w & \text{if } (m,w) \text{ are engaged.} \end{cases}$$

Init: Current[m] = -1 ∀m } O(n)

Query: Read current[m] ← O(1)

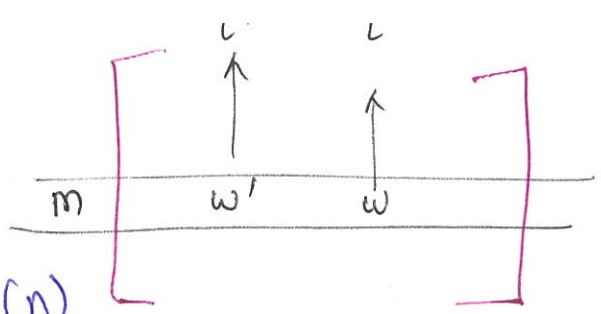
Update: After w's proposal, (m, w) get engaged → current[m] = w ← O(1)

So far: Init : O(n)+O(n)+O(n) = O(n) : Query/update: O(1)

Q4) Is $w' > w$ in L_m?

A4') Scan ManPref[m] and figure out ranks i' and i of women w' & w in L_m
→ check is $i' < i$?
↑ O(1)

} O(n)



ManPref
↳ overall GS runtime $O(n^3)$.