## Lecture 21

CSE 331
Oct 21, 2022

## Project deadlines coming up

| Fri, Oct 28 | Counting Inversions $\mathbf{D}^{F 21} \mathbf{D}^{F 19} \mathbf{D}^{F 18} \mathrm{D}^{F 17} \mathrm{x}^{2}$ | [KT, Sec 5.3] (Project (Problem 1 Coding) in) |
| :---: | :---: | :---: |
| Mon, Oct 31 | Multiplying large integers $\mathbf{D}^{F 21} \mathbf{D}^{F 19} \mathbf{D}^{\text {P18 }} \mathbf{D}^{F 17} \mathrm{x}^{2}$ | [ KT, Sec 5.5] (Project (Problem 1 Reflection) in) |
|  |  | Reading Assignment: Unraveling the mystery behind the identity |
| Wed, Now 2 | Closest Pair of Points $\mathbf{D}^{\text {P21 }} \boldsymbol{\square}^{819} \boldsymbol{D}^{-18} \boldsymbol{D}^{[17} \mathrm{x}^{2}$ | [ $\mathrm{KT}, \mathrm{Sec} 5.4$ ] |
| Fri, Nov 4 |  | [KT, Sec 5.4] (Project (Problem 2 Coding ) in) |
| Mon, Nov 7 | Weighted interval Scheduling $\mathbf{D}^{F 21} \mathbf{D}^{F 19} \mathbf{D}^{F 17} \mathrm{x}^{2}$ | [ KT, Sec 6.1] (Project (Problem 2 Reflection) in) |

## Group formation instructions

## Autolab group submission for CSE 331 Project

The lowdown on submitting your project (especially the coding and reflection) problems as a group on Autolab.

Follow instructions


The instruction below are for Coding Problem 1
You will have to repeat the instructions below for EACH coding AND reflection problem on project on Autolab (with the appropriate changes to the actual problemp)

## Form your group on Autolab

Groups on Autolab will NOT be automatically created
You will have to form a group on Autolab by yourself (as a groupl. Aead on for indituctions on how to go about this.

## Mid-term temp grade assigned

metame 0 \# 6.

## Mid-term temp grade






" Lel Qle rae anar 1 toin liad el a mas el in

- Lat M be pour mid-tens woos inut of a mas al 1M0.



```
R=\frac{N}{B}\cdotN+Q\cdot\frac{1}{H}+\frac{R}{B}\cdotM.
```


#  




- Aerape 21.1
a Mediar tis.
a Dad Dec H1 H?
- Max al


## 1-on-1 meetings

## Meetings to discuss CSE 331 performance

 Areqhe.






 44ber





#### Abstract







```
moluer frusy,
```


## Questions/Comments?



## Minimum Spanning Tree Problem

Input: Undirected, connected $G=(V, E)$, edge costs $c_{e}$
Output: Subset $\mathrm{E}^{\prime} \subseteq \mathrm{E}$, s.t. $\mathrm{T}=\left(\mathrm{V}, \mathrm{E}^{\prime}\right)$ is connected $C(T)$ is minimized

If all $c_{e}>0$, then $T$ is indeed a tree

## Kruskal's Algorithm

Input: $G=(V, E), c_{e}>0$ for every e in $E$

$$
\mathrm{T}=\varnothing
$$

Sort edges in increasing order of their cost

Consider edges in sorted order


Joseph B. Kruskal

If an edge can be added to $T$ without adding a cycle then add it to $T$

## Prim's algorithm

Similar to Dijkstra's algorithm


Robert Prim

Input: $\mathrm{G}=(\mathrm{V}, \mathrm{E}), \mathrm{c}_{\mathrm{e}}>0$ for every e in E
$S=\{s\}, T=\varnothing$
While $S$ is not the same as $V$
Among edges $e=(u, w)$ with $u$ in $S$ and $w$ not in $S$, pick one with minimum cost
Add w to $S$, e to $T$

## Cut Property Lemma for MSTs

Condition: $S$ and $V \backslash S$ are non-empty


Cheapest crossing edge is in all MSTs

Assumption: All edge costs are distinct

## Questions/Comments?



## Today's agenda

Optimality of Prim's algorithm
Prove Cut Property Lemma
Optimality of Kruskal's algorithm

Remove distinct edge weights assumption

## On to the board...



## Optimality of Kruskal's Algorithm

Input: $G=(V, E), c_{e}>0$ for every e in $E$
$T=\varnothing$
Sort edges increasing order of their cost
$S$ is non-empty
$V \backslash S$ is non-empty
First crossing edge considered

Consider edges in sorted order
If an edge can be added to without adding a cycle chen add it to $T$

## Is ( $\mathrm{V}, \mathrm{T}$ ) a spanning tree?

No cycles by design

Just need to show that $(\mathrm{V}, \mathrm{T})$ is connected


