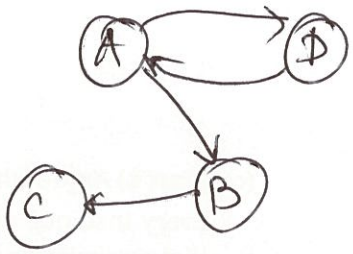
Sep 21

$G = (V, E)$ $\quad E \subseteq V \times V$

V: set of vertices / nodes

E: set of edges

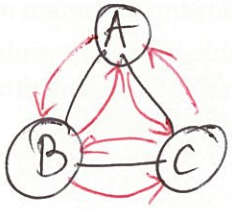


$V = \{A, B, C, D\}$

$E = \{(A,D), (D,A), (A,B), (B,A), \ldots\}$

$n = 4$

$m = 4$



$V = \{A, B, C\}$

$E = \{(A,B), (B,C), (C,A), (B,A), (C,B), (A,C)\}$

$n = 3$

$m = 3 (6)$

Default: $|V| = n$, $|E| = m$

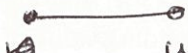
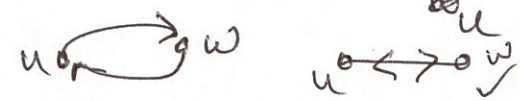
Def: G is undirected

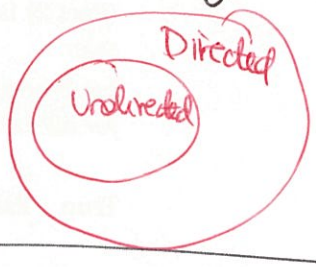
$\iff \forall\ u \neq w \in V,\ (u,w) \in E \iff (w,u) \in E$

Q: Airline map (undirected) **Q':** Wikipedia articles

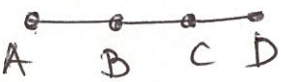
Default: G is undirected

Claim: Every undirected graph is also a directed graph

Pf (idea): Replace every  by 



Paths



A ✓

D, C, B, A ✓

A, B, C, D ✓

A, B, C, B ✓

A, C, D ✗

D, C, B, A ✗

A, B, C, D ✓ A ✓

A, B, C, B ✗

A, C, D ✗

Def: A path in a ~~directed~~ (directed) graph $G=(V,E)$ is a sequence of $k \geq 1$ vertices $u_1, \ldots, u_k$ s.t. $\forall i \in [k-1]$ $(u_i, u_{i+1}) \in E$

$[k-1] = \{1, \ldots, k-1\}$

Note: (i) u_i need not be distinct (ii) holds for directed graphs

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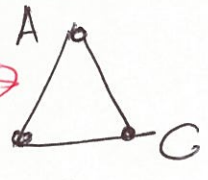
Def: A simple path is a path with no repeated vertices

Ex: Any simple path length $\leq n-1$

Def: The length of a path is the # edges in path.

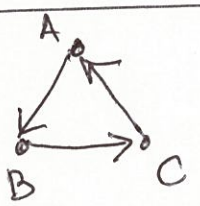
Cycles

Triangle graph →



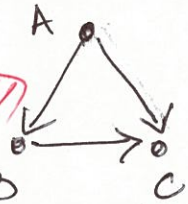
A, B, C, A ✓

A, C, B, A ✓



A, B, C, A ✓

A, C, B, A ✗



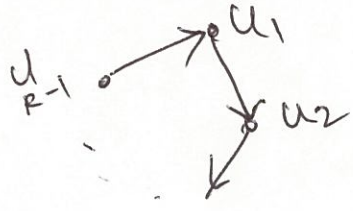
A, B, C, A ✗

A, C, B, A ✗

Directed Acyclic graph (DAG)

Def: A cycle is a sequence $u_1, u_2, \ldots, u_k (= u_1)$ $u_1, \ldots, u_{k-1}$ are distinct

s.t. $\forall i \in [k-1]$ $(u_i, u_{i+1}) \in E$ Condition on k $u \circlearrowright u$



(i) Directed graph $k \geq 3$

(ii) Undirected graph $k \geq 4$