Sep 23

PROPOSITION: Let T be a BFS tree for $G=(V,E)$

If $(u,w) \in E$ s.t $u \in L_i$, $w \in L_j$

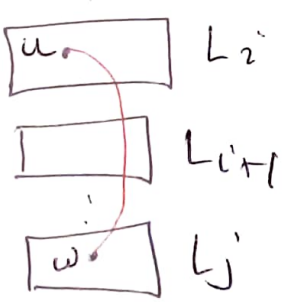
$\Rightarrow |i-j| \leq 1 \iff i \in \{j-1, j, j+1\}$

Pf (idea) By contradiction

WLOG (Without loss of generality) assume $i \leq j$ [if $i > j$ switch the roles of i & j in the pf. below]

For contradiction assume $|i-j| > 1 \Rightarrow j \geq i+1$

$\Rightarrow j \geq i+2$



[S] L_0 Consider the time when BFS was creating L_{i+1}

(•) $u \in L_i$, $w \notin L_0, \ldots, L_i$

(•) $(u,w) \in E$

$\Rightarrow$ w will be added to L_{i+1} (by definition of BFS)

$\rightarrow$ contradicts $w \in L_j$ for $j \geq i+2$ ∎

Explore (s)

0. $R \leftarrow \{s\}$
1. while $\exists (u,w) \in E$ s.t $u \in R, w \notin R$
   Add w to R
2. Output $R^* \leftarrow R$