

Oct 3

# Interval Scheduling Problem

[3, 8]

= {3, 4, 5, 6, 7}

Input:  $n$  intervals:  $i^{\text{th}}$  interval  $[s(i), f(i)]$

start time

= {s(i), f(i)-1}

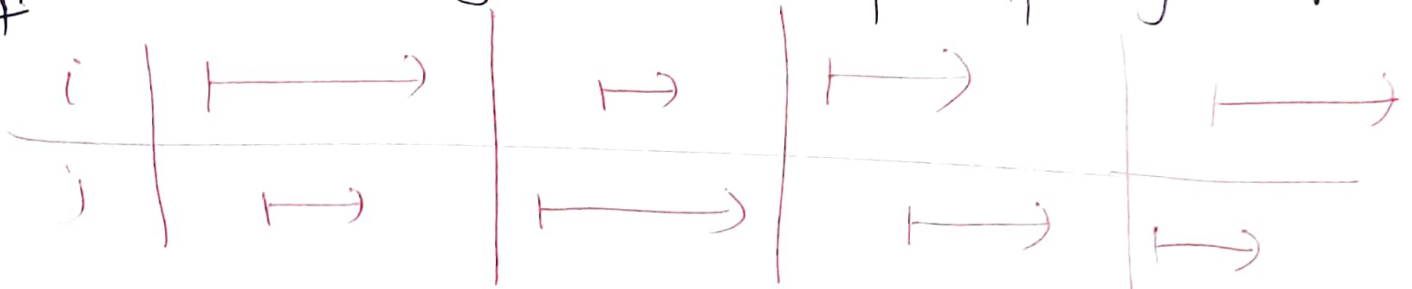
finish time  
 $f(i)-1$

Output: A valid schedule with max # intervals in it  
 $\uparrow$  over all valid schedules

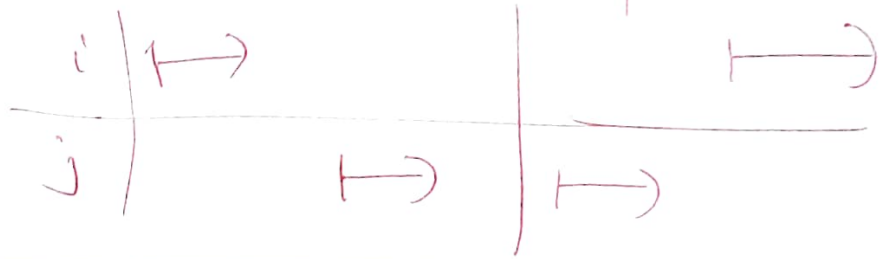
Def: A schedule  $S \subseteq [n]$  def  $\{1, \dots, n\}$  <sup>schedules</sup>

Def: A valid schedule  $S$  has no conflicts

Def: Intervals  $i$  &  $j$  have a conflict if they overlap



$\Rightarrow$  no conflict



Obs: A valid schedule sorted by start/finish time gives the same order.

Assume: Input intervals are sorted by finish time

$$f(1) \leq f(2) \leq \dots \leq f(n)$$

$\uparrow$  If not sort in  $O(n \log n)$  time

# Greedy Algo

0.  $R \leftarrow [n]$

1.  $S \leftarrow \phi$

2. While  $R \neq \phi$

(2.1) let  $i$  be the smallest index in  $R$

(2.2) Add  $i$  to  $S$

(2.3) Remove  $i$  from  $R$

(2.4) Delete all  $j \in R$  that conflict w/  $i$

3. Return  $S^* \leftarrow S$

OK since  
 $f(1) \leq f(2) \leq \dots \leq f(n)$

---

THM: