Oct 14

Last Fri: Special case $l_e = 1 \quad \forall e \in E$

→ run HW3 Q3

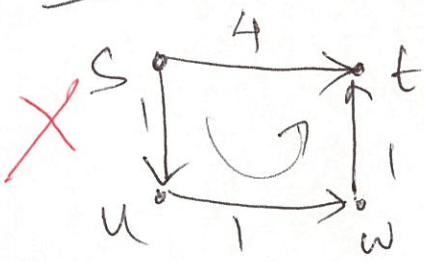
($l_e = L > 0 \; \forall e \in E$ for the same L)

Algo idea: Reduce general case to $l_e = 1 \; \forall e \in E$

$l_e \geq 0 \quad \forall e \in E$

$G \; \{l_e\} \longrightarrow G' \; \{l'_e = 1\}$

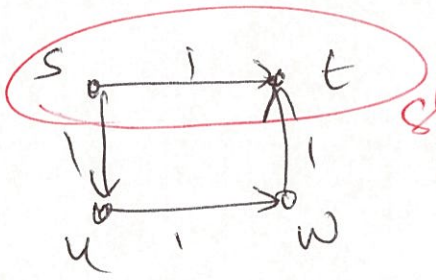
Idea 1: Ignore all edge lengths



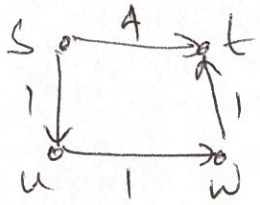
X

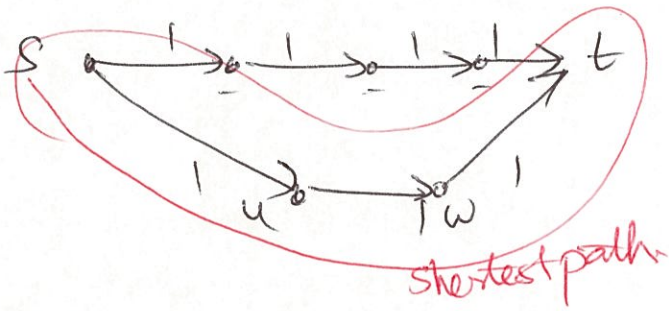
$d(t)$ ⇒

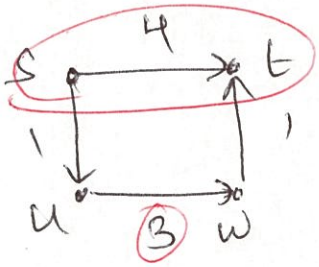
ignore all l_e



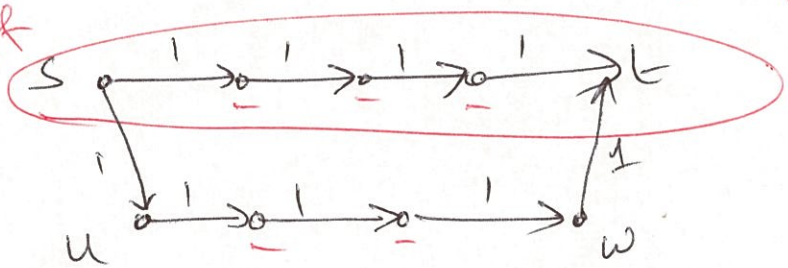
Harrison's idea:







shortest path



Algo idea: $G \longrightarrow G'$ Replace each edge $e \in E$ with a path of length l_e in G'

$\{l_e\} \rightarrow \{l'_e = 1\}$

(new paths do not share any edges)

(Ex)

Claims: A shortest s-t path in G

$\Leftrightarrow$ equiv shortest s-t path in G'

⇒ Run HW3 Q3 algo on G'

Correctness: Claim + Algo for HW3 Q3 is correct

$G' = (V', E')$

runtime = $O(|V'| + |E'|)$

Ex: Reduction for $G \to G'$ can be done in $O(|V'| + |E'|)$

$n' = |V'|$, $m' = |E'|$

$\Rightarrow O(n' + m')$ $\ell_{max} = \max_{e \in E} \ell_e$ $\quad n \le n' \le \ell_{max} \cdot n$, $\quad m \le m' \le \ell_{max} \cdot m$

$\le O(\ell_{max}(n+m))$ If $\ell_{max} \le O(1) \Rightarrow$ ✓

What if $\ell_{max} = n^{100}$ $\Rightarrow O(n^{100}(n+m))$ time!

Aside: RAM model ⇒ unit of space is a register

If you have n items in your input register have $O(\log n)$ bits.

⇒ Adjacency list $O(m+n)$ registers

Q: How many register do you need to represent $\le n^{100}$

⇒ $\approx 100 \log n$ bits $O(\log n)$

$\approx$ 100 registers or $O(1)$ registers

⇒ $O(m+n)$ registers to store graph with $\ell_{max} \le n^{O(1)}$

IDEALLY: $O(m+n)$ runtime for