Oct 17

Lem 1: At the end of each iteration, if $u \in R$, path P_u is a shortest s-u path

↑ s-u path in Dijkstra tree.

Pf (idea) By induction on $|R|$

Base case: $|R| = 1 \Rightarrow R = \{s\}$, $d(s) = 0$ ✓

I.H.: Assume lemma statement is true for $|R| = k$ for some $k \geq 1$

I.S. Assume $|R| = k+1$

Let w be the $(k+1)^{th}$ vertex added to R

$\Rightarrow P_w = P_u, (u,w)$ of Dijkstra tree

Assume w was discovered via $u \in R$

$\Rightarrow d(w) = d(u) + \ell(u,w)$

Goal: Show P_w is a shortest s-w path.

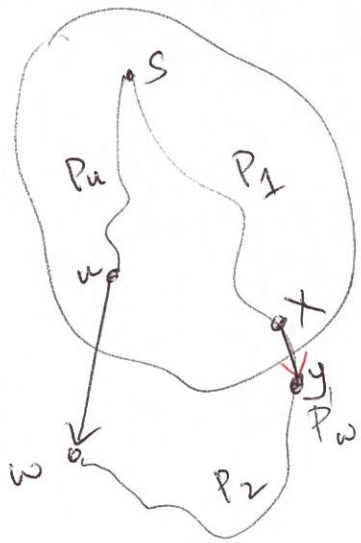
Pf (idea) By contradiction.

(Think of state of algo just before w is added to R)

Assume $\exists$ s-w path $P'_w \neq P_w$ but

$\ell(P'_w) < \ell(P_w)$ ——— (*)

Since $s \in R$ but $w \notin R$ $\Rightarrow P'_w$ has to "cross" R.



$P'_w = P_1, (x,y), P_2$

$\ell(P'_w) = \ell(P_1) + \ell(x,y) + \ell(P_2)$

$\geq d(x) + \ell(x,y) + \ell(P_2)$ — since $d(x)$ is distance to x in s

$\geq d'(y)$

$\geq d'(y) + \ell(P_2) \geq d'(y)$ — $\ell(P_2) \geq 0$ as $\ell e \geq 0$ $\forall e$

$\geq d'(w) = d(w) = \ell(P_w)$ — Algo chose w over y

$\Rightarrow \ell(P'_w) \geq \ell(P_w)$ → contradicts (*) ∎