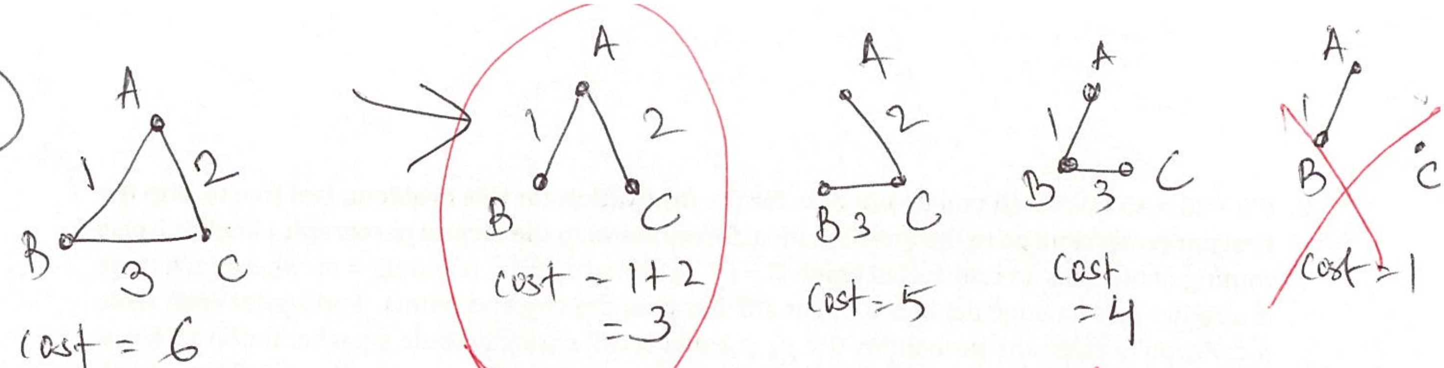
Oct 19



Input: $G = (V, E)$, $\forall e \in E$ $c_e \geq 0$

connected

undirected

cost

convenience only

Output: $E' \subseteq E$ s.t. (spanning) subgraph of

(i) $T = (V, E')$ is connected

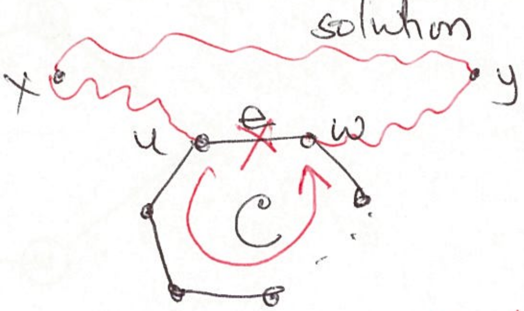
(ii) $c(T) = \sum_{e \in T} c_e$ is minimized

<u>M</u>inimum <u>S</u>panning <u>T</u>ree (MST)

PROP: Let $c_e > 0$ $\forall e \in E$, then any optimal solution $T = (V, E')$ is a tree.

Pf (idea) By contradiction. Let T be an optimal solution (*)

but T is NOT a tree



$\Longrightarrow$ $\exists$ a cycle C

as T is connected

→ Let e be any edge in C

Goal: Show another ~~tre~~ $T' = (V, E'')$ s.t. $c(T') < c(T)$

spanning subgraph

set difference

→ Delete e from T, $T' = (V, E' \setminus \{e\})$

Claim 1: $c(T') < c(T)$. $c(T') = c(T) - c_e$

as $c_e > 0$ → $< c(T)$

Claim 2: T' is still connected

Let $x \neq y \in V$

~~Case 2~~ Case 2.1: $\exists$ x-y path that does not use e.

Case 2.2: All x-y paths use e

$\Rightarrow$ Use rest of cycle to connect u to w

$\Rightarrow$ x, y are still connected in T'!

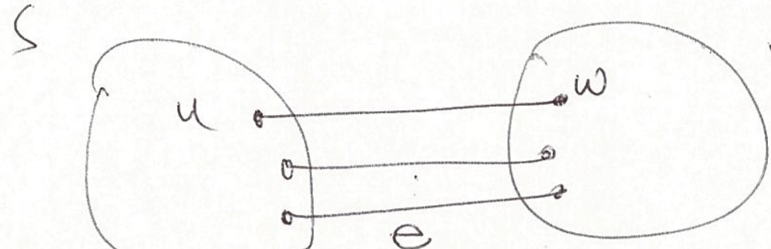
T' is a spanning subgraph (Claim 2) but by Claim 1 $c(T') < c(T)$

$\Rightarrow$ T is not optimal $\Rightarrow$ contradicts (*)

CUT PROPERTY LEMMA

Assume: All c_e's are distinct ← (remove this later)

for all cuts $(S, V \setminus S)$ s.t. $S \neq \emptyset$ and $V \setminus S \neq \emptyset \equiv V \neq S$



Consider all "crossing edges" $(u,w) \in E$, $u \in S$, $w \notin S$

Let e be the crossing edge w/ min cost.

$\Longrightarrow$ e is in ALL MSTs for G.

Idea: For both Prim / Kruskal every edge added is the cheapest crossing edge for some cut.