Nov 7

Weighted Interval Scheduling

Input: n intervals

i^{th} interval $i \in [n] = (s_i, f_i, v_i)$

$[n] = \{1, \ldots, n\}$

start time (s_i), finish time (f_i), value (v_i)

Output: Instead of outputting an optimal solution $\mathcal{O}$,

output $v(\mathcal{O}) = \sum_{i \in \mathcal{O}} v_i$

Def: $OPT(j)$ = value of any optimal solution $[j]$, $j \in [n]$

(s_1, f_1, v_1)

$\vdots$

(s_j, f_j, v_j)

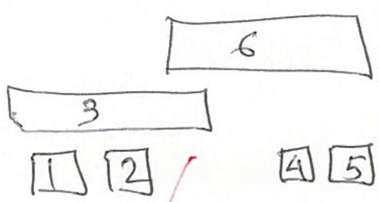
$f_1 \le f_2 \le \cdots \le f_n$

Q: Goal? A: $OPT(n)$

Def: Let $\mathcal{O}_j$ be an optimal schedule for $[j]$

$v(\mathcal{O}_j) = OPT(j)$

Ex: $j = 6$



Case: $j \notin \mathcal{O}_j$

Ex $6 \notin \mathcal{O}_6$

Claim: $\mathcal{O}_6$ (Ex) is an optimal soln for $[5]$

Ex: $\mathcal{O}_j$ is opt for $[j-1]$

$\Rightarrow$ $\boxed{OPT(j) = OPT(j-1)}$

(Ex) Claim: $\mathcal{O}_6 \setminus \{6\}$ is opt soln for $\{1, 2\}$

Case 2: $j \in \mathcal{O}_j$ $\quad 6 \in \mathcal{O}_6$

$OPT(6) = v_6 + OPT(2)$

anything the conflicts w/ $6/j$ is out

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Def: $p(j)$ to be the largest index $i < j$ s.t. i & j do not conflict

$p(6) = 2$ $\quad \boxed{OPT(j) = v_j + OPT(p(j))}$

$OPT(j) = \max\{OPT(j-1), v_j + OPT(p(j))\}$