

Sep 7

Stable Matching

$n = 2$
 $M = \{BP, BBT\}$
 $W = \{JA, AJ\}$

→ Perfect matching

Preference lists:

$L_{JA} : BP > BBT$
(BP, BBT)

$L_{AJ} : BP > BBT$

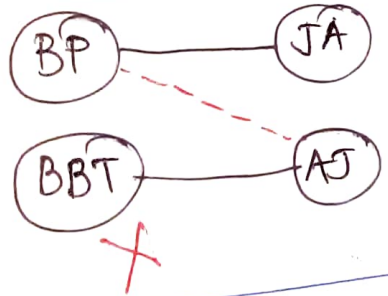
$L_{BP} : AJ > JA$

$L_{BBT} : AJ > JA$

Def (Preference list) : $\forall w \in W, L_w$: Total ranking of all n men
 $\forall m \in M, L_m$: Total ranking of all n women

general n : (Q1) How many preference lists are there?
(Q2) How many elements across all pref. lists?
A: $2n \times n = 2n^2 = \Theta(n^2)$

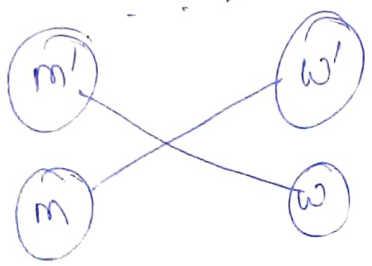
Stable matching



Def : A stable matching is
(i) a perfect matching
(ii) with NO instability

Def (Instability) Given $2n$ pref list
AND a perfect matching $S \subseteq M \times W$

a pair (m', w') is an instability IF



(1) $m' > m$ in $L_{w'}$

AND

(2) $w' > w$ in $L_{m'}$

Stable Matching Problem:

Input : (i) M, W
(ii) $L_w \forall w \in W, L_m \forall m \in M$

Output : Stable matching (if it exists)