Sep 9

Gale-Shapley algo

① Initialize all men & women to be free

② In a loop: book: men proposes.

A free woman proposes to a man

(stuff happens)

③ You have n matched pairs

Initial state: all n men + women are free

① Let w be a free woman

Q1: Which man m should w propose to?

A1: The man m on top of L_w

w proposes to m

(Q2) What should m do?

Accept? X

Reject? X

m should get engaged to w

General state: A man/woman is either free/engaged.

Case 1: All n men & n women are engaged

→ Algo terminates

Case 2: A free woman w

Q3: Who should w propose to?

A3: To the best man m that she has not proposed to yet?

Sep 13 m proposes to w

Case 2.1: m is free ⇒ (m, w) get engaged

Case 2.2 m is engaged to w'

Running Example

n=2, M={BP, BBT}, W={JA, AJ}

L_{AJ}: BBT > BP	L_{BP}: JA > AJ
L_{JA}: BP > BBT	BBT: AJ > JA

AJ	JA	BP	BBT
F	(F)	F	F

Free

Q1: Who should JA propose to?

A: BP

(JA proposes → BP)

Q2: What should BP do?

Accept? X

Reject? X

(BP, JA) get engaged

AJ	JA	BP	BBT
(F)	E	E	F

Engaged

Q3: Who should AJ propose to?

A3: BBT

(AJ → BBT) proposed

Q4: What should BBT do?

(AJ, BBT) get engaged

AJ	JA	BP	BBT
E	E	E	E

Case 2.1: w' > w in L_m ⇒ no change

Case 2.2: w > w' in L_m w' is free

THEOREM: For every input (n, M, W, 2n pref lists) the GS outputs a stable matching (!) $|M| = |W| = n$

⇒ COROLLARY: Every input to the stable matching problem has a stable matching.

Pf: follows from THEOREM.

Pf of THEOREM

→ Say S is the o/p of the GS algo on an arbitrary input.

Want to argue: S is a stable matching

Lemma 1: For every i/p, GS terminates

Lemma 2: S is a perfect matching

Lemma 2: S has NO instability

Lemma 1+2+3 ⇒ THM.