## Lecture 20

CSE 331 Oct 16, 2023

## Exams this week

Mid-term 1: Wednesday

Mid-term 2: Friday

## **Project Group formation**

📕 note @331 💿 ★ 🔒 -

autolab

project



good note 0 Edit Updated 21 hours ago by Atri Rudra

# My response to your feedback

📄 note @336 💿 ★ 🔓 -

#### Reponse to feedback

Thanks to everyone who give feedback @281!

Below, I will post some pie-charts that I think give some interesting overall picture of how y'all feel about the course and then some responses to the written comments. I apologize for the delay in doing this and I understand that some of this feedback could have been useful if given earlier-- sorry about that 😒

First some pie-charts:

Overall your feeling about CSE 331

#### 27 responses



While of course having unhappy/very unhappy students is not ideal, at least the fraction of students who are very unhappy are (comfortably larger) than those that are not very unhappy. This was **not** the case in the few couple of offerings so I'm glad to see the "tide turn" time around. Also (bit less than) 50% of the respondents are not unhappy. Again not ideal but better than where this was few course offerings ago.



Actions -

## Questions?



## Shortest Path problem



Output: Length of shortest paths from s to all nodes in V

## Towards Dijkstra's algo: part ek

Determine d(t) one by one





## Towards Dijkstra's algo: part do

Determine d(t) one by one

Let u be a neighbor of s with smallest  $\ell_{(s,u)}$ 



Not making any claim on other vertices



Length of  $\checkmark$  is  $\geq 0$ 

## Towards Dijkstra's algo: part teen

#### Determine d(t) one by one

Assume we know d(v) for every v in R

Compute an upper bound d'(w) for every w not in R

$$d(w) \leq d(u) + \ell_{(u,w)}$$

- $d(w) \leq d(x) + \ell_{(x,w)}$
- $d(w) \leq d(y) + \ell_{(y,w)}$



 $d'(w) = \min_{e=(u,w) \text{ in } E, u \text{ in } R} d(u) + \ell_e$ 

## Questions/Comments?



## Dijkstra's shortest path algorithm



d(w) = d'(w)

 $d'(w) = \min_{e=(u,w) \text{ in } E, u \text{ in } R} d(u) + \ell_e$ 

d(u) = 1

d(x) = 2

d(z) = 4

## Questions/Comments?



## Couple of remarks

The Dijkstra's algo does not explicitly compute the shortest paths

Can maintain "shortest path tree" separately

Dijkstra's algorithm does not work with negative weights

Left as an exercise

## Rest of Today's agenda

Prove the correctness of Dijkstra's Algorithm

## Dijkstra's shortest path algorithm

P<sub>u</sub> shortest s-u path in "Dijkstra tree"

 $d'(w) = \min_{e=(u,w) \text{ in } E, u \text{ in } R} d(u) + \ell_e$ 

Input: Directed G=(V,E),  $\ell_e \ge 0$ , s in V

R = {s}, d(s) =0 While there is a x not in R with (u,x) in E, u in R Pick w that minimizes d'(w) Add w to R d(w) = d'(w)

Lemma 1: At end of each iteration, if u in R, then P<sub>u</sub> is a shortest s-u path

Lemma 2: If u is connected to s, then u in R at the end

## Proof idea of Lemma 1



## Dijkstra's shortest path algorithm

 $d'(w) = \min_{e=(u,w) \text{ in } E, u \text{ in } R} d(u) + \ell_e$ 

 $\Sigma_{x \in V} O(In_x+1)$ =O(m+n) time

Input: Directed G=(V,E),  $\ell_e \ge 0$ , s in V

 $R = {s}, d(s) = 0$ 

Add w to R

d(w) = d'(w)

While there is a x not in R with (u,x) in E, u in R

Pick w that minimizes d'(w)

At most n iterations

O((m+n)n) time bound is trivial

O((m+n) log n) time implementation with priority Q

## **Reading Assignment**

Sec 4.4 of [KT]

