

# Lecture 22

CSE 331

Oct 25, 2023

# Project deadlines coming up

Tue, Oct 31		(HW 5 in)
Wed, Nov 1	Multiplying large integers  F22  F21  F19  F18  F17 x <sup>2</sup>	[KT, Sec 5.5] <i>Reading Assignment:</i> <a href="#">Unraveling the mystery behind the identity</a>
Fri, Nov 3	Closest Pair of Points  F22  F21  F19  F18  F17 x <sup>2</sup>	[KT, Sec 5.4] (Project (Problems 1 & 2 <b>Coding</b> ) in)
Mon, Nov 6	Kickass Property Lemma  F22  F21  F19  F18  F17 x <sup>2</sup>	[KT, Sec 5.4] (Project (Problems 1 & 2 <b>Reflection</b> ) in)

# Group formation instructions

## Autolab group submission for CSE 331 Project

The lowdown on submitting your [project](#) (especially the [coding](#) and [reflection](#)) problems as a group on Autolab.

Follow instructions **EXACTLY** as they are stated

**The instructions below are for Coding Problem 1**

You will have to repeat the instructions below for EACH coding AND reflection problem on project on Autolab (with the appropriate changes to the actual problem).

## Form your group on Autolab

**Groups on Autolab will NOT be automatically created**

You will have to form a group on Autolab by yourself (as a group). Read on for instructions on how to go about this.

# HW 5 out

## Homework 5

Due by **11:30pm, Tuesday, October 31, 2023.**

Make sure you follow all the [homework policies](#).

All submissions should be done via [Autolab](#).

Check the [week 9 recitation notes](#) for this homework.

## Question 1 (Computing Set Intersection on a Network) [50 points]

### The Problem

In this problem, we will take a break from trying to minimize the runtime of the algorithm and focus on an important resource in distributed computing: the total number of bits communicated over a network by the algorithm.

### ! For those of you who are feeling a little ambitious

For the top 3 submissions in the scoreboard in Python, the top 2 submissions in the scoreboard in Java and the top submission in the scoreboard in C++, we are offering 2.5 bonus points. But be warned! You should not be spending too much time on this. We rather you work on Questions 1 and 2 above.

# I finished grading all Q3(b)s

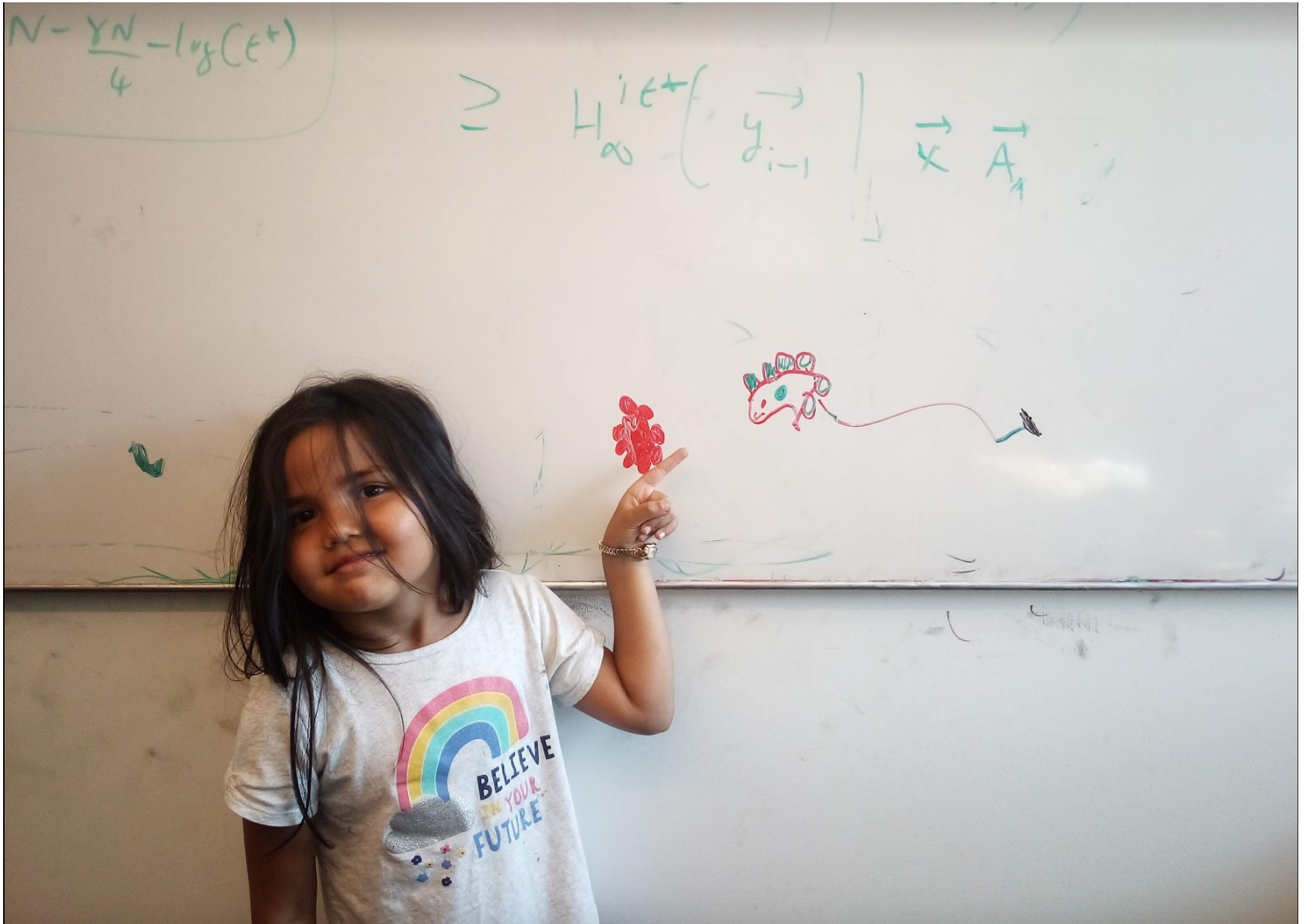
Mid-term grades should be out by  $\leq$  tomorrow afternoon

It was not pretty.....

And I don't get why many of you just left Q3(b) blank!

You get better by struggling and not giving up!

# Questions/Comments?



# Minimum Spanning Tree Problem

**Input:** Undirected, connected  $G = (V, E)$ , edge costs  $c_e$

**Output:** Subset  $E' \subseteq E$ , s.t.  $T = (V, E')$  is connected  
 $C(T)$  is minimized

If all  $c_e > 0$ , then  $T$  is indeed a tree

# Kruskal's Algorithm

Input:  $G=(V,E)$ ,  $c_e > 0$  for every  $e$  in  $E$

$T = \emptyset$

Sort edges in increasing order of their cost

Consider edges in sorted order

If an edge can be added to  $T$  without adding a cycle then add it to  $T$



Joseph B. Kruskal



# Prim's algorithm

Similar to Dijkstra's algorithm



Robert Prim

Input:  $G=(V,E)$ ,  $c_e > 0$  for every  $e$  in  $E$

$S = \{s\}$ ,  $T = \emptyset$

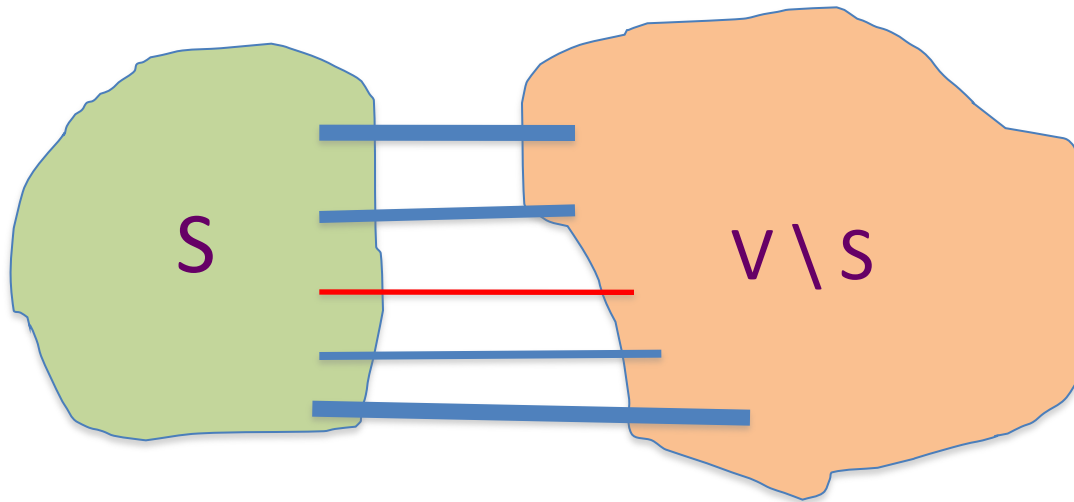
While  $S$  is not the same as  $V$

Among edges  $e = (u,w)$  with  $u$  in  $S$  and  $w$  not in  $S$ , pick one with minimum cost

Add  $w$  to  $S$ ,  $e$  to  $T$

# Cut Property Lemma for MSTs

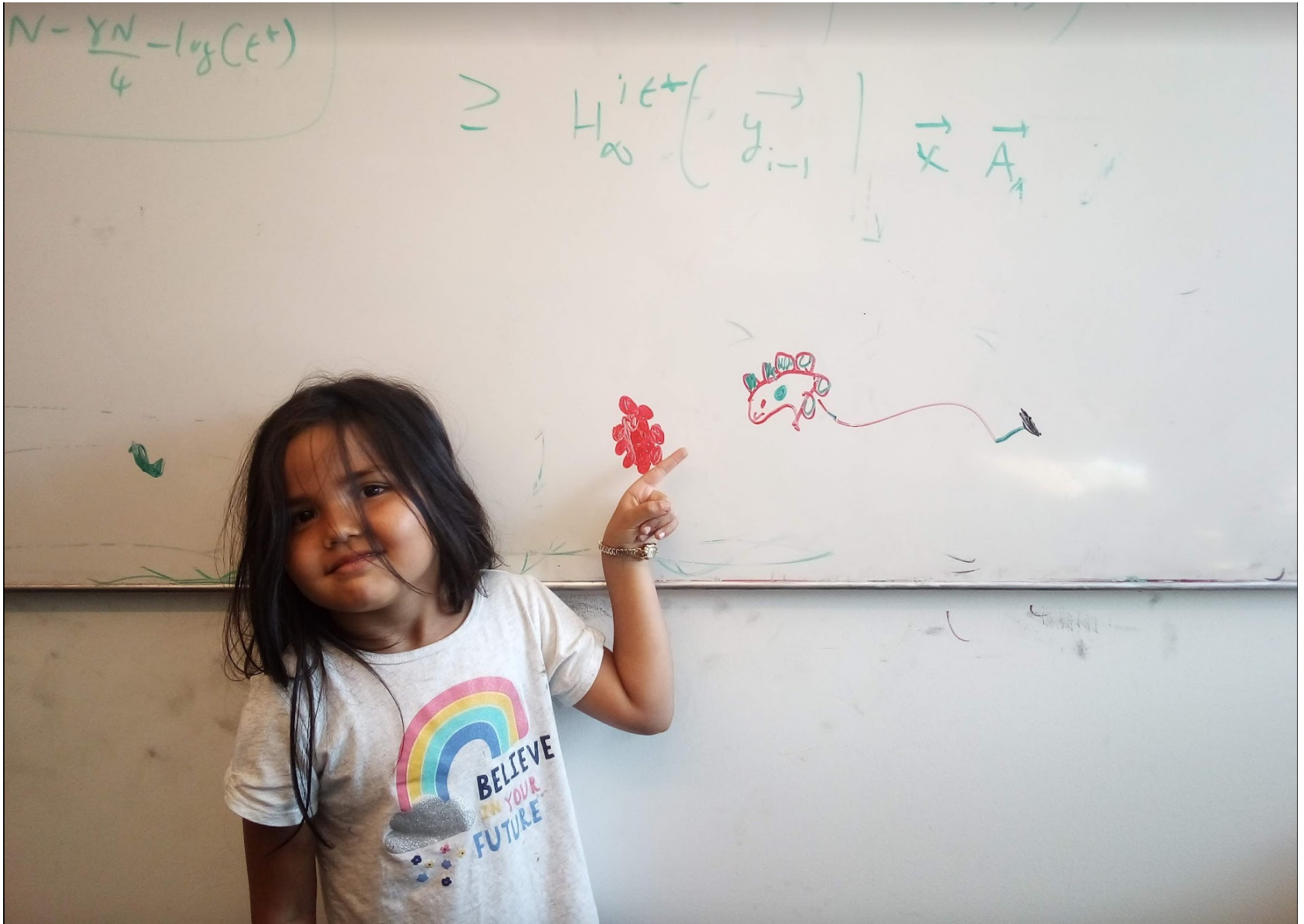
Condition:  $S$  and  $V \setminus S$  are non-empty



Cheapest crossing edge is in **all** MSTs

Assumption: All edge costs are distinct

# Questions/Comments?



# Today's agenda

Optimality of Prim's algorithm

Prove Cut Property Lemma

Optimality of Kruskal's algorithm

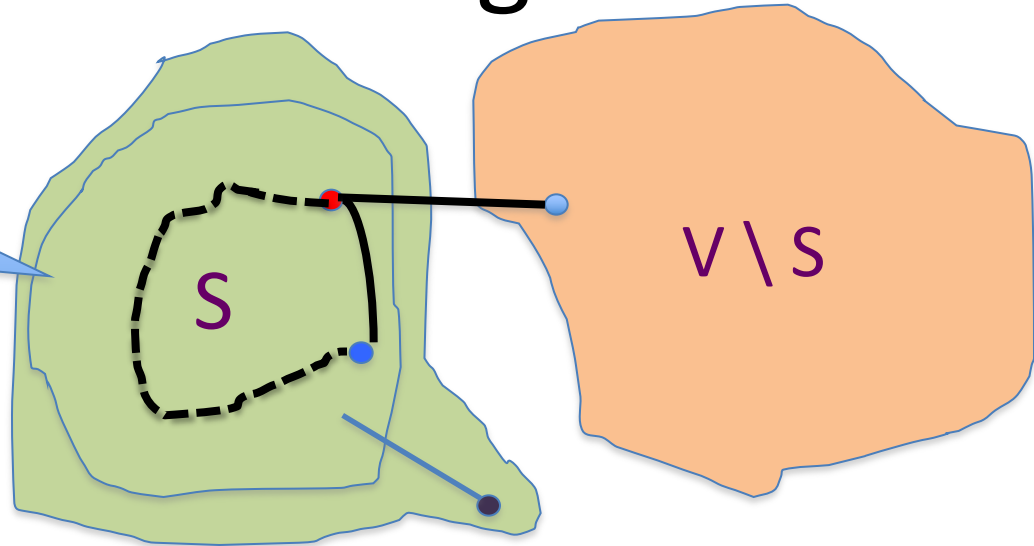
Remove distinct edge weights assumption

# On to the board...



# Optimality of Kruskal's Algorithm

Nodes connected to red in  $(V, T)$



Input:  $G=(V,E)$ ,  $c_e > 0$  for every  $e$  in  $E$

$T = \emptyset$

Sort edges in increasing order of their cost

Consider edges in sorted order

If an edge can be added to  $T$  without adding a cycle then add it to  $T$

$S$  is non-empty

$V \setminus S$  is non-empty

First crossing edge considered