

Sep 25

PROPOSITION: Let T be a BFS tree for $G=(V,E)$

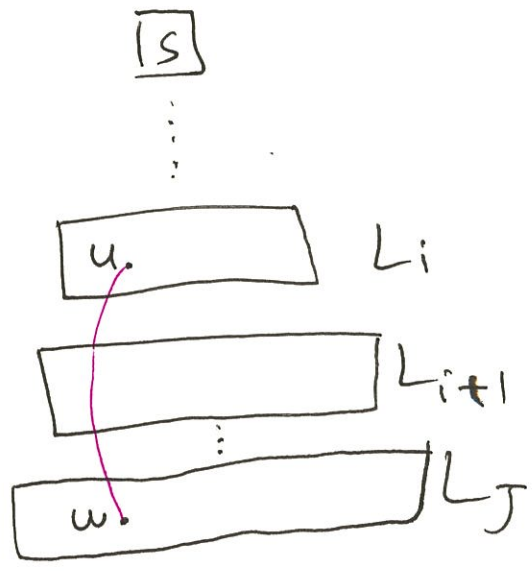
If $(u,w) \in E$ s.t. $u \in L_i, w \in L_j$

$\Rightarrow |i-j| \leq 1 \Leftrightarrow i \in \{j-1, j, j+1\}$

Pf(idea) By contradiction

WLOG assume $i \leq j$ [if $i > j$ switch the roles of i & j in the pf below]
without loss of generality

For contradiction assume $|i-j| > 1 \Rightarrow j > i+1$
 $j \geq i+2$



Consider the time when BFS was creating L_{i+1}

- $u \in L_i, w \notin L_0, \dots, L_i$
- $(u,w) \in E$

$\Rightarrow w$ will be added to L_{i+1} (by def of BFS)
 \rightarrow contradicts $w \in L_j$ for $j \geq i+2$