$\boxed{\text{Oct 4}}$

Interval Scheduling Problem

$[3,8) = \{3, \cancel{5}4, 5, 6, 7\}$

Input: n intervals : i^{th} interval $[s(i), f(i))$

$1 \le i \le n$

start time

finish time

$f(i)-1$

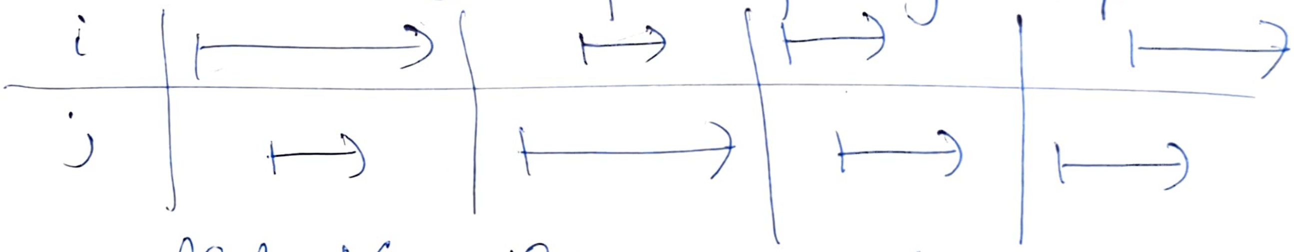
$= \{s(i), \ldots, f(i)-1\}$

Output: A valid schedule with the max number of intervals in it.

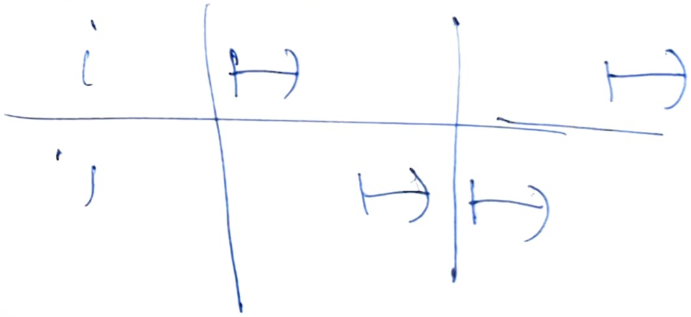
Def: A schedule $S \subseteq [n] \overset{\text{def}}{=} \{1, \ldots, n\}$

Def: A valid schedule S has no conflicts

Def: Intervals i & j conflict if they overlap



⇒ no conflict b/w i & j



Claim: A valid schedule that is sorted by start time or by finish time gives the same order

Assume: Input intervals are sorted by their finish times

$f(1) \le f(2) \le f(3) \le \cdots \le f(n)$

↳ If not: in $O(n \log n)$ time sort the intervals.

Greedy algo

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0.  $R \leftarrow [n]$
1.  $S \leftarrow \phi$
2.  While $R \neq \phi$
        (2.1) Let $i$ be the smallest index in $R$        OK since $f(1) \leq f(2) \leq \cdots \leq f(n)$
        (2.2) Add $i$ to $S$
Combine ~~(2.3)~~ Remove $i$ from $R$
        (2.4) Delete all $j \in R$ (from $R$) that conflict with $i$
3.  Return $S^* \leftarrow S$
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Theorem 1: S^* is an optimal solution → ∀ inputs, among all possible valid schedules for that input, S^* has the max number of intervals.

Pf. of correctness
- → Greedy stays ahead (next)
- → Exchange argument (min. max lateness → See 4.2)