Oct6

Greedy algo

0. $R \leftarrow [n]$
1. $S \leftarrow \phi$
2. While $R \neq \phi$

   (2.1) Let i be the ~~s~~smallest index in R ← OK since $f(1) \leq f(2) \leq \cdots \leq f(n)$

   (2.2) Add i to S

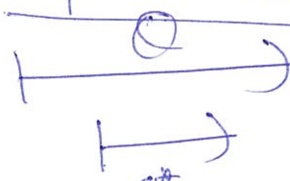
   Combine { ~~(2.3)~~ Remove i from R
   (2.4) Delete all $j \in R$ (from R) that conflict with i }
3. Return $S^* \leftarrow S$

Theorem 1: S^* is an optimal solution → $\forall$ inputs, among all possible valid schedules for that input, S^* has the max number of intervals.

Pf. of correctness → Greedy stays ahead (next)
→ Exchange argument (min. max lateness → See 4.2)

Pf (idea) of Thm 1: Let O be an optimal solution

Idea: $O = S^*$ ✗ since more than 1 opt solution is possible.



THEOREM 2: $|S^*| = |O|$

($\Rightarrow$ S^* is also optimal since O is optimal by def.)

Notation: $S^* = \{i_1, \ldots, i_k\}$ $f(i_1) \leq f(i_2) \leq \cdots \leq f(i_k)$

$O = \{j_1, \ldots, j_m\}$ $f(j_1) \leq f(j_2) \leq \cdots \leq f(j_m)$

THEOREM 3: $k = m$

Claim 1: $k \leq m$ {Pf (idea): Since O is optimal}

THEOREM 4: $k \geq m$ (as this + Claim 1 $\Rightarrow$ THM 3)

THM 4: $k \geq m$

$S^* = \{ i_1, \ldots, i_k \}$ $[k \leq m]$

$\mathcal{O} = \{ j_1, \ldots, j_m \}$

Lemma 1: (Greedy Stays Ahead) $\forall$ $1 \leq l \leq k$

$$f(i_l) \leq f(j_l)$$

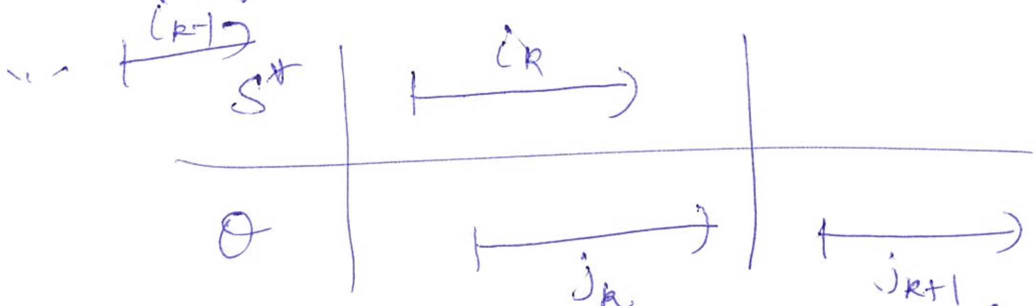
[Assume Lemma 1 is true]

Pf (idea) of THM 4: Proof by contradiction

Assume $k < m \Rightarrow m \geq k+1 \Rightarrow j_{k+1} \in \mathcal{O}$

By Lemma 1, $f(i_k) \leq f(j_k)$

Consider the situation right after i_k is picked by greedy algo



Recall: Algo has terminated at this point — (*)

Claim: (EX) $j_{k+1} \in R$

(As j_{k+1} doesn't conflict with $i_k, i_{k-1}, \ldots$)

$\Rightarrow R \neq \phi$ at the end $\xRightarrow[\text{algo defn}]{}$ Greedy algo cannot have terminated. contradicts ∎