Oct 11

Lemma 1: $\forall \; 1 \le l \le k,$

$$f(i_l) \le f(j_l)$$

$S^* = \{i_1, \dots, i_k\}$, $f(i_1) \le \dots \le f(i_k)$

$O = \{j_1, \dots, j_m\}$

$f(j_1) \le \dots \le f(j_m)$

Pf (sketch): Induction on l. since $f(1) \le f(2) \le \dots \le f(n)$

Base case: $l = 1$, $f(i_1) = f(1) \le f(j_1)$ ✓

by defn of greedy algo

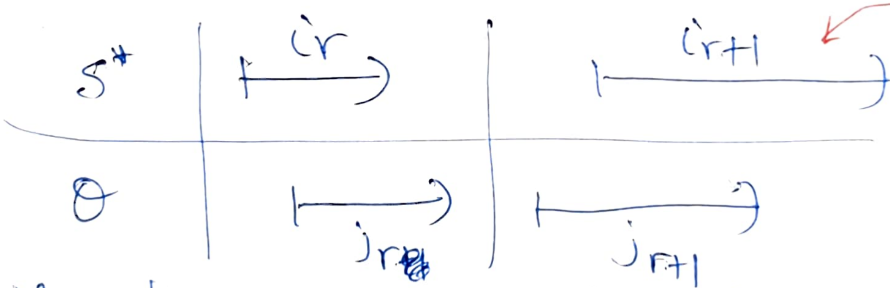
Inductive Hypothesis Assume for some $k > r \ge 1$, $\forall \; 1 \le l \le r$

$f(i_l) \le f(j_l)$ $\Rightarrow i_{r+1} \in S^*$ — (#)

Inductive step: Argue $f(i_{r+1}) \le f(j_{r+1})$.

$\rightarrow$ Pf by contradiction. Assume $f(i_{r+1}) > f(j_{r+1})$

By I.H. $f(i_r) \le f(j_r)$



Consider the scenario just after i_r was added to S by greedy algo.

$\Rightarrow i_{r+1}, j_{r+1} \in R$

But since $f(j_{r+1}) < f(i_{r+1}) \Rightarrow$ greedy cannot pick i_{r+1} in next iteration.

contradicts the fact that $i_{r+1} \in S^*$ (#) $\Leftarrow$

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Shortest path problem

Input: A directed graph $G = (V, E)$

$s \in V$

'lengths' $l_e \geq 0 \quad \forall e \in E$

integer

needed for Dijkstra

Output: $\forall t \in V$, output a shortest s-t path

$$l(P) = \sum_{e \in P} l_e$$

distance from s to t in G

Simpler version: Output $d(t) \quad \forall t \in V$

length of any shortest s-t path.

Special case: $l_e = 1 \quad \forall e \in E$

Use HW3 Q3 algo.

Also solves $l_e = L \quad \forall e \in E$
