

Qdrt

$O(1) \rightarrow$ If $n=1$ then return a_1

floor $\lfloor 0.3 \rfloor = 0$
ceil $\lceil 0.3 \rceil = 1$

$O(n) \left\{ \begin{array}{l} a_L = a_1 \dots a_{\lfloor \frac{n}{2} \rfloor} \\ a_R = a_{\lfloor \frac{n}{2} \rfloor + 1} \dots a_n \end{array} \right.$

$T(\lfloor \frac{n}{2} \rfloor)$

return MERGE (MergeSort ($a_L, \lfloor \frac{n}{2} \rfloor$), MergeSort ($a_R, n - \lfloor \frac{n}{2} \rfloor$)) $T(n - \lfloor \frac{n}{2} \rfloor)$

$T(n)$ def max number of MergeSort over ALL inputs of size n .

Thm 1 $T(n)$ is $O(n \log n)$

$$\begin{aligned} \frac{T(n)}{n=1} &\leq O(1) + O(n) + T(\lfloor \frac{n}{2} \rfloor) + T(n - \lfloor \frac{n}{2} \rfloor) + O(n) \\ T(1) &= O(1) \end{aligned}$$

$$\begin{aligned} T(n) &\leq O(n) + T(\lfloor \frac{n}{2} \rfloor) + T(n - \lfloor \frac{n}{2} \rfloor) \\ &\leq O(n) + T(\lfloor \frac{n}{2} \rfloor) + T(\lceil \frac{n}{2} \rceil) \\ &\leq O(n) + T(\frac{n}{2}) + T(\frac{n}{2}) \\ &= O(n) + 2T(\frac{n}{2}) \end{aligned}$$

$$T(n) = \begin{cases} O(1) & \text{if } n=1 \\ O(n) + 2T(\frac{n}{2}) & \text{if } n>1 \end{cases}$$