Nov 9

Closest-in-Box (S) ← returns closest pair $(p, p') \in S$ if $d(p, p') < \delta$ & null otherwise

$S = \{ (x,y) \in P \mid |x - x^*| < \delta \}$

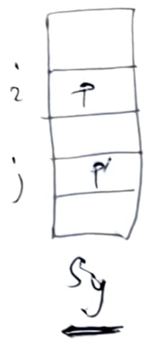
KICKASS PROPERTY LEMMA

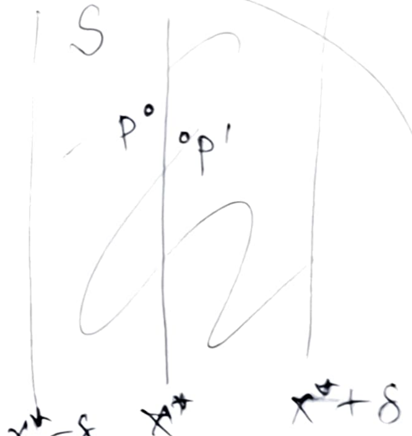
For every $p \neq p' \in S$ s.t. $d(p,p') < \delta$

if $p = S_y[i]$

$p' = S_y[j]$

then $|i - j| \leq 15$ (!)





$|x - x^*| < \delta$

$\iff x^* - \delta < x < x^* + \delta$

Note: Can replace "15" by 9 (or even 7)

Q: $O(n)$ algo for Closest-in-box (assuming the)

$n' = |S|$

$O(n)$ for $i = 1 \ldots n'$

check $(S_y[i], S_y[i+1])$

$O(1)$ $(S_y[i], S_y[i+2])$, ..., $(S_y[i], S_y[i+15])$ ← min $(i+15, n')$

let (p_i, p_i') be the closest pair of points from above

$O(n)$ → Let (p, p') be the closest pair among (p_i, p_i') $i = 1 \ldots n'$

If $d(p, p') < \delta$
return (p, p')
else
null

$O(1)$

Overall: $O(n)$

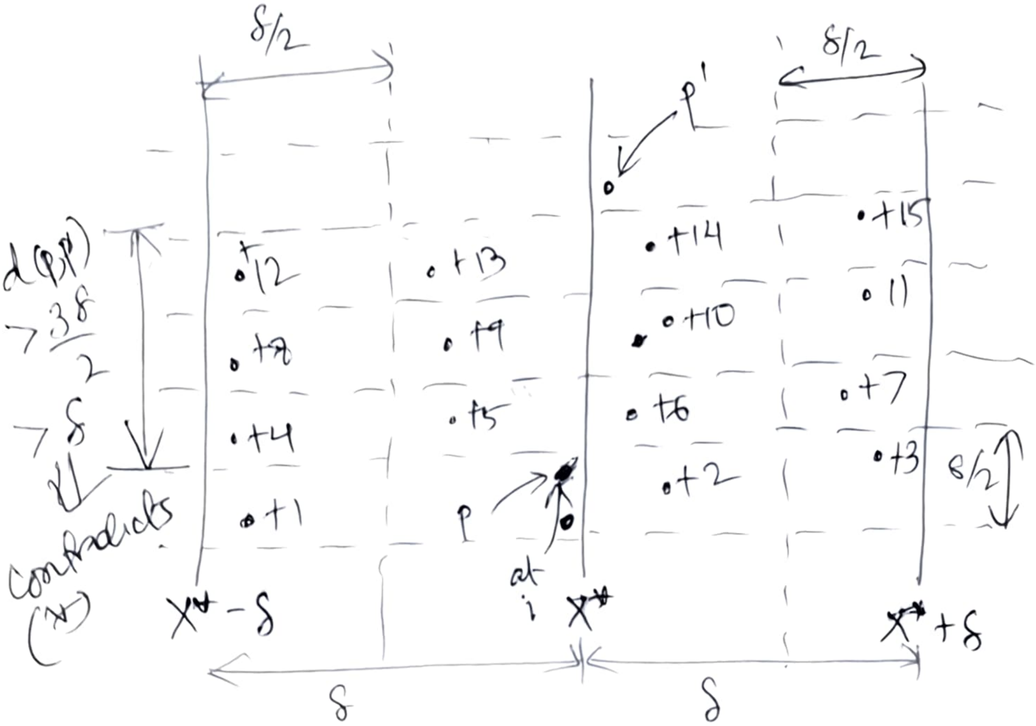
Pf (idea) of the Kickass Property lemma

By contradiction: Assume $\exists p \neq p' \in S$ s.t. $d(p, p') < \delta$ — (*)

s.t. $p = S_y[i]$, $p' = S_y[j]$ where $j \geq i + 16$

$i, i+1, \ldots, i+15, | i+16$

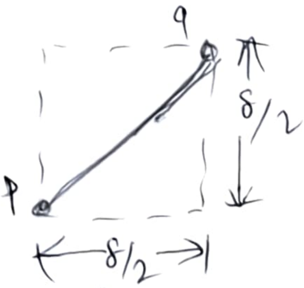
$\uparrow p$ $\qquad \rightarrow p'$



<u>Claim:</u> Every $\frac{\delta}{2} \times \frac{\delta}{2}$ square boxes have at most 1 point from S in it

<u>Pf (idea)</u> By contradiction. Assume $\exists p \neq q \in S$ s.t. p & q are in one square

Ex) p & q are farthest apart if they are on corners of a diagonal



$$d(p, q) \leq \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2} = \sqrt{\frac{\delta^2}{4} + \frac{\delta^2}{4}} = \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}}$$

as $\sqrt{2} > 1 \rightarrow < \delta$

As each square is completely within Q or R

$\Rightarrow$ 2 pts in Q / R with dist $< \delta$

$\hookrightarrow$ contradicts the def of δ.