Weighted Interval Scheduling

(Nov 10)

Input: n intervals, i^{th} interval (s_i, f_i, v_i), $i \in [n] = \{1, \dots, n\}$

(s_i = start time, f_i = finish time, v_i = value)

Output: A valid schedule $S \subseteq [n]$ s.t. its value $v(S) \overset{def}{=} \sum_{i \in S} v_i$ is maximized.

(Note: Interval Scheduling problem: $v_i = 1 \; \forall i$
$\Rightarrow v(S) = |S|$)

Output': Instead of outputting an optimal soln O, output its value $v(O)$

Def: $OPT(j)$ = value of an optimal solution for sub-problem $[j]$, $1 \le j \le n$

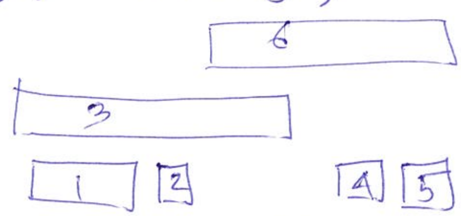
(s_1, f_1, v_1)
$\vdots$
(s_j, f_j, v_j)

Assume: $f_1 \le f_2 \le \dots \le f_n$

Q: Goal? A: $OPT(n)$

Def: $j \in [n]$, let O_j be an optimal soln for $[j]$
$\Rightarrow v(O_j) = OPT(j)$

Ex: $n = 6$



Case 1: $j \notin O_j$ Ex: $6 \notin O_6$ ⟶

Claim 1: O_6 is a valid solution for $[5]$

(EX) Claim 2: O_6 is an optimal solution for $[5]$

(EX) Claim 3: $j \notin O_j \Rightarrow O_j$ is opt for $[j-1]$

$OPT(j) \quad OPT(j-1)$)

<u>Case 2:</u> $j \in O_j$ Ex: $6 \in O_6$

$6 \in O_6 \longrightarrow$ [1] [2]

$OPT(6) = v_6 + OPT(2)$

Ex: $OPT(6) = \max\{OPT(5),\ v_6 + OPT(2)\}$