Nov 15

Assume: (i) have access to $p(1), \ldots, p(n)$
(ii) $f_1 \le f_2 \le \ldots \le f_n$
} Can do both in $O(n\log n)$ time

Compute $M[0..n]$

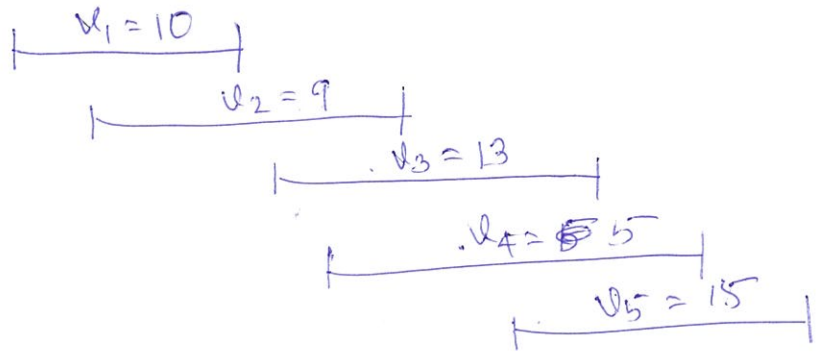
① $M[0] \leftarrow 0$

② for $j = 1..n$

$M[j] \leftarrow \max\{v_j + M[p(j)], M[j-1]\}$

③ return ~~$M[n]$~~ MSchedule $(n; M, p)$

$n=5$



$p(1) = 0$
$p(2) = 0$
$p(3) = 1$
$p(4) = 1$
$p(5) = 2$

$n=5$

	0	1	2	3	4	5
$j=0$	0					
$j=1$	0	10				
$j=2$	0	10	10			
$j=3$	0	10	10	23		
$j=4$	0	10	10	23	23	
$j=5$	0	10	10	23	23	25

$M[0] \leftarrow 0$

$M[1] = \max\{v_1 + M[p(1)], M[0]\}$
$= \max\{10 + 0, 0\} = 10$

$M[2] = \max\{v_2 + M[0], M[1]\}$
$= \max\{9 + 0, 10\} = 10$

$M[3] = \max\{v_3 + M[1], M[2]\}$
$= \max\{13 + 10, 10\} = 23$

$M[4] = \max\{v_4 + M[1], M[3]\}$
$= \max\{5 + 10, 23\} = 23$

$M[5] = \max\{v_5 + M[2], M[4]\}$
$= \max\{15 + 10, 23\} = 25$

$\Rightarrow \boxed{OPT(5) = 25}$

$O_5 = \{1, 5\}$

Recall: $j \in O_j \iff v_j + \underbrace{OPT(p(j))}_{M[p(j)]} > \underbrace{OPT(j-1)}_{M[j-1]}$

can replace by $\geq$

$\boxed{n=5}$ $5 \overset{?}{\in} O_5$

$v_5 + M[p(5)] \overset{?}{>} \cancel{OPT}\, M(4)$

$15 + 10 \overset{?}{>} 23$ ✓ $\Rightarrow 5 \in O_5$

$p(5) = 2 \Rightarrow O_5 \setminus \{5\}$ over $[2]$
$= O_2$

$2 \overset{?}{\in} O_2$

$9 + 0 \overset{?}{>} 10$ X $\Rightarrow 2 \notin O_2$

We have $O_2 = O_1$

$1 \overset{?}{\in} O_1$ $10 + 0 \overset{?}{>} 0$ ✓ $\Rightarrow 1 \in O_1$

$\Rightarrow O_5 = \{1, 5\}$

MSchedule $(n; M, p)$

If $n = 0$ return ϕ

If $v_n + M[p(n)] > \cancel{OPT(n}\, M[n-1]$

return $\{n\} \cup$ MSchedule $(p(n); M, p)$

else return MSchedule $(n-1; M, p)$

$O(n)$ time

SUBSET SUM problem

Output a subset of the 3 jobs so that the sum of w_i's chosen $\leq W$ & max the sum of w_i's

Ex: $n = 3$ $w_1 = 1$, $w_2 = 3$, $w_3 = 3$

Budget W

(i) $W = 7$, opt $= \{1, 2, 3\}$

(ii) $W = 6$, opt $= \{2, 3\}$

(iii) $W = 5$, opt $= \{1, 2\}$ or $\{1, 3\}$

In general the sum of w_i's in opt $< W$

Input: n integers $w_1, \ldots, w_n$ s.t $w_i > 0$ and a budget W

Output: A subset $S \subseteq [n]$ s.t

(1) $\sum_{i \in S} w_i \leq W$ (2) max $w(S) = \sum_{i \in S} w_i$