

Sep 8

Stable matching

$n = 2$

$W = \{JA, AJ\}$

$M = \{BP, BBT\}$

→ Perfect matching ✓

Pref lists:

$L_{JA} : BP > BBT$
 (BP, BBT)
 $L_{AJ} : BP > BBT$

$L_{BP} : AJ > JA$
 $L_{BBT} : AJ > JA$

$n \geq 1$

Def (Preference list)

$\forall w \in W, L_w : \text{Total ranking of all } n \text{ men}$

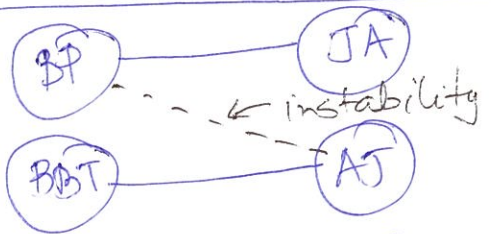
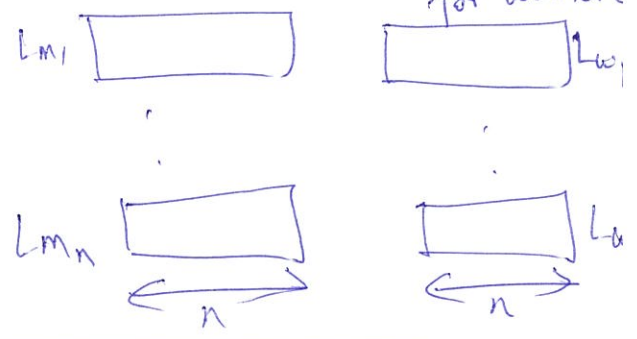
$\forall m \in M, L_m : \text{Total ranking of } n \text{ women}$

general n: (Q1) How many pref lists in total? $2n$

(Q2) How many elements in total across all lists?

Ans: $2n$ pref list
 $\Rightarrow 2n \times n = 2n^2 = \Theta(n^2)$

Input size



Def: A stable matching is

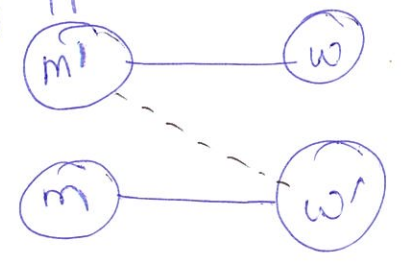
- (i) a perfect matching
- (ii) with NO instability

Def (instability)

perfect matching $S \subseteq M \times W$, a pair $(m', w') \notin S$ is an instability IF

(i) $m' > m$ in $L_{w'}$
~~AND~~ AND

(ii) $w' > w$ in $L_{m'}$



Stable Matching problem

Input:

- (i) $M, W; n$
- (ii) $L_w \forall w \in W$
 $L_m \forall m \in M$

Output: A stable matching (if it exists)