

Sep 11 Gale-Shapley algo

- Initialize all n men and n women to be free
- In a loop: \leftarrow Book: men propose
A free woman proposes to a man
.... stuff happens....
- We have n matched pairs

Initial state: All n men + n women are free

- Let w be a free woman
- Q1: Which man m should w propose to?
A1: The man m in top of L_w
 $\rightarrow w$ proposes to m
- Q2: What should m do?
Accept \Rightarrow might have better offer
Reject \Rightarrow might NOT get a better proposal later.
 $\rightarrow (m, w)$ get engaged

General state: A man/woman is either free/engaged.

Case 1: All n men & n women are engaged.
 \rightarrow Algo terminates & output n matched pairs

- Case 2: \exists a free woman w
- Q3: Who should w propose to?
A3: To the best man m that w has not proposed to
 $\rightarrow w$ proposes to m
- Q4: What should m do?

Running Example

$n=2$ $M = \{BP, BBT\}$, $W = \{JA, AJ\}$

$L_{AJ}: BBT > BP$	$L_{BP}: JA > AJ$		
$L_{JA}: BP > BBT$	$L_{BBT}: AJ > JA$		
AJ	JA	BP	BBT
F	(F)	F	F

- Q1: Who should JA propose to?
A1: BP



- Q2: What should BP do?
Accept? (BP, JA) get engaged
Reject?

AJ	JA	BP	BBT
(F)	E	E	F

AJ is free

- Q3: Who should AJ propose to?
A3: BBT
AJ $\xrightarrow{\text{propose}}$ BBT

- Q4: What should BBT do?
(BBT, AJ) get engaged

AJ	JA	BP	BBT
E	E	E	E

Case 2.1 m is free $\Rightarrow (m, w)$ get engaged

Case 2.2 (m, w') are engaged

Case 2.2.1 $w' > w$ in $L_m \Rightarrow$ do nothing

Case 2.2.2 $w > w'$ in $L_m \Rightarrow (m, w)$ get engaged
 w' is free.
