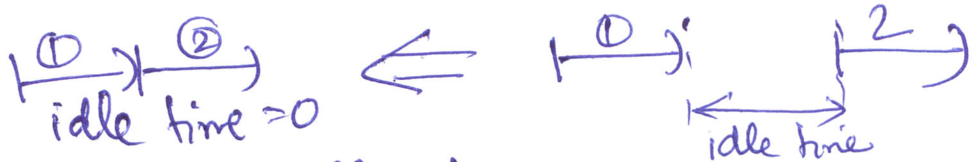
Oct 13

Let S be schedule output by greedy algo.
Let Θ be an optimal schedule
[Recall: $L(S)$ is max lateness of schedule S)

THM 1: $L(S) = L(\Theta)$

Def: Idle time of a schedule is max gap between any 2 consecutively scheduled jobs.



Obs 1: S has 0 idle time

Obs 2: Can assume Θ has 0 idle time.
(If not "squish" the gaps $\Rightarrow$ finish times $f(i)$ can only decrease $\Rightarrow$ $l_i = \max(0, f(i)-1-d_i)$ can only decrease $\Rightarrow$ $L(\Theta)$ doesn't change.)

Def: Given a schedule S, a pair of jobs (i,j) is an inversion IF (1) ~~d~~ $d_i > d_j$ AND (2) i is scheduled before j in S.

Obs 3: S has 0 # ~~inv~~ inversions

LEMMA 1: If S_1 & S_2 have BOTH 0 idle time and 0 # inversion $\Rightarrow$ $L(S_1) = L(S_2)$

LEMMA 2: S has 0 idle time & 0 # inversions.

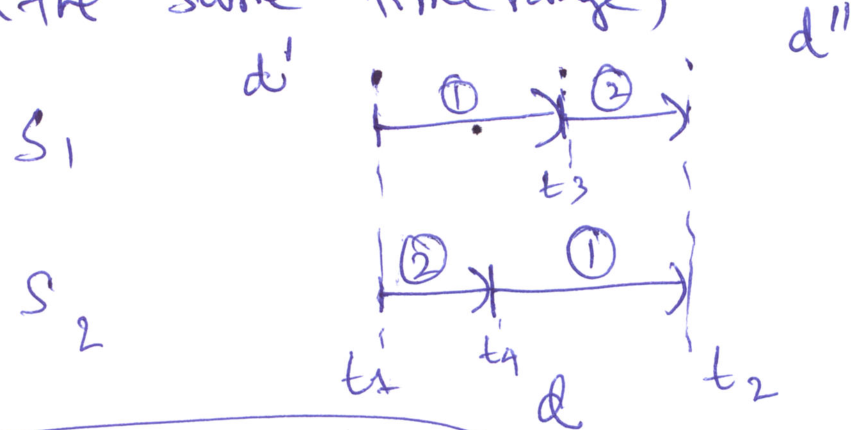
LEMMA 3: $\exists$ an optimal schedule Θ s.t. it has 0 idle time & 0 # inversions.

Lemmas 1+2+3 $\Rightarrow$ THM 1.

(Pf idea of Lemma 1) Recall: S_1 & S_2 both have 0 idle time and 0 # inversions.

<u>Claim:</u> For any schedule w/ 0 idle time & 0 # inversions, for any deadline d, all jobs i s.t. $d_i = d$ are scheduled right next to each other (in the same time range)

$d' < d < d''$
↳ follows as there are 0 # inv.



$\boxed{\text{Assume Claim is true}}$ Analyze l_1, l_2

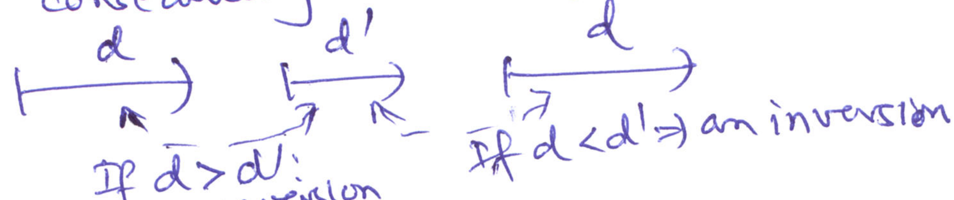
<u>In S_1:</u>
$l_1 = \max(0, t_3 - 1 - d)$
$l_2 = \max(0, t_2 - 1 - d)$
$\max(l_1, l_2) = l_2 = \max(0, t_2 - 1 - d)$

<u>In S_2:</u>
$l_1 = \max(0, t_2 - 1 - d)$
$l_2 = \max(0, t_4 - 1 - d)$
$\max(l_1, l_2) = l_1 = \max(0, t_2 - 1 - d)$

$\Rightarrow$ max lateness among all jobs w/ deadline d, does not change $\Rightarrow$ (as choice of d was arbitrary) $L(S_1) = L(S_2)$ ∎

<u>Pf idea of Claim:</u> 0 # inversions $\Rightarrow$ all jobs with same deadline are consecutively scheduled

If not



0 idle time $\Rightarrow$ no gap between 2 consecutive jobs

(Use induction to finish argument ∎