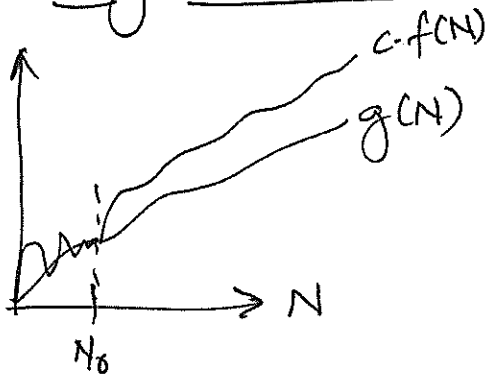
Sep 12

Big-Oh notation:

$g(N)$ is $O(f(N))$ if $\exists\ c>0$, $N_0>0$



$10N+1 \stackrel{?}{=} O(\sqrt{N})$ ✗

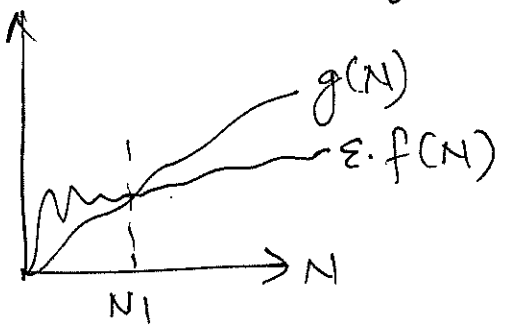
$= O(N)$ ✓

$= O(N^2)$ ✓

$= O(N!)$ ✓

Big-Omega notation:

$g(N)$ is $\Omega(f(N))$ if $\exists\ \varepsilon>0$, $N_1>0$



$10N+1 = \Omega(\sqrt{N})$ ✓

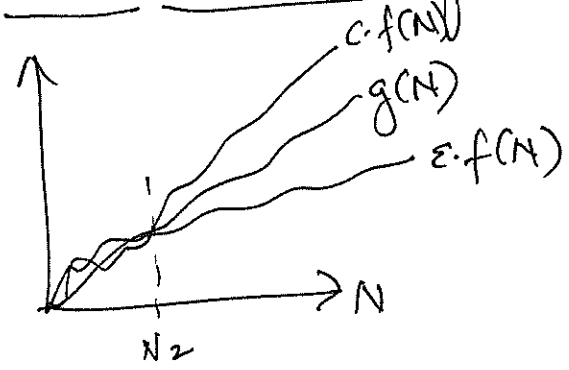
$= \Omega(N)$ ✓

$= \Omega(N^2)$ ✗

$= \Omega(N!)$ ✗

Big-Theta notation:

$g(N)$ is $\Theta(f(N))$ if $g(N)$ is $O(f(N))$ AND $g(N)$ is $\Omega(f(N))$



$10N+1 \stackrel{?}{=} \Theta(\sqrt{N})$ ✗

$= \Theta(N)$ ✓

$= \Theta(N^2)$ ✗

$= \Theta(N!)$ ✗

Properties of O (also valid for Ω, Θ)

① Transitive: $g(N)$ is $O(f(N))$ AND $f(N)$ is $O(h(N))$

$\Rightarrow g(N)$ is $O(h(N))$

Ex: $10N+1$ is $O(N)$ and N is $O(N^2) \Rightarrow 10N+1$ is $O(N^2)$

② Additive: $f(N)$ is $O(h(N))$, $g(N)$ is $O(h(N))$

$\Rightarrow f(N)+g(N)$ is $O(h(N))$

Ex: $10N$ is $O(N)$ and 1 is $O(N) \Rightarrow 10N+1$ is $O(N)$

③ $f(N)$ is $O(h_1(N))$ and $g(N)$ is $O(h_2(N))$

$\Rightarrow f(N) \cdot g(N)$ is $O(h_1(N) \cdot h_2(N))$

$\rightarrow$ N is $O(N)$, 10 is $O(1)$ $\Rightarrow 10 \cdot N = O(N \cdot 1) = O(N)$